Dynamic behaviour of ancient freestanding multi-drum and monolithic columns subjected to horizontal and vertical excitations - ScienceDirect

LINK TO PUBLISHER VERSION:

https://www.sciencedirect.com/science/article/abs/pii/S0267726118311941

DOI:

https://doi.org/10.1016/j.soildyn.2019.01.024

Dynamic behaviour of ancient freestanding multi-drum and monolithic columns subjected to horizontal and vertical excitations

V. Sarhosis[1,2*], D. Baraldi[3], J.V. Lemos[4], G. Milani[5]

(1) University of Leeds, Leeds, UK
(2) Newcastle University, Newcastle upon Tyne, UK
(3) Università IUAV di Venezia, Venice, Italy
(4) National Laboratory for Civil Engineering, Lisbon, Portugal
(5) Technical University of Milan, Milan, Italy

Abstract

Earthquakes represent one of the major threats to cultural heritage monuments, such as classical ancient columns. Understanding the behaviour and dynamic response of such historic structures is useful for the assessment of the conservation and rehabilitation techniques to be used for their preservation. The behaviour of ancient multi-drum and monolithic columns subjected to dynamic loads is characterised by highly nonlinearity since both rocking and sliding phenomena can occur. Analytical studies of multi-drum columns subjected to dynamic load is extremely complicated, if not impossible to perform. Nowadays, computational methods of analysis can be used to represent their dynamic response. Using a software based on the Discrete Element Method (DEM) of analysis, a typical ancient multi-drum and an equivalent in dimensions monolithic columns subjected to horizontal and combined horizontal and vertical harmonic excitations were modelled to identify the main factors affecting their stability. Different acceleration amplitude and frequency input records were applied and their role in the collapse/deformation mechanism was investigated. From the results analyses it was shown that novel structural analysis tools that extend traditional methods of structural assessment could allow engineers to understand the mechanisms that have allowed the surviving structures to avoid structural collapse and destruction during strong earthquakes.

Keywords: multi-drum column, monolithic column, ground motion, DEM, dynamic response, structural analysis

[*] **Corresponding author:** Dr Vasilis Sarhosis, School of Engineering,Newcastle University, Newcastle upon Tyne, UK, email: vasilis.sarhosis@newcastle.ac.uk

1. Introduction

Natural events such as earthquakes represent a major threat to cultural heritage structures including ancient temples and towers (Croci 1998a; Croci 1998b; Macchi 1998). Ancient temples consist of multi-drum or monolithic columns made of marble or limestone. Multi-drum columns were constructed by placing each drum (or block of stone) on top of each other. Thus, during shaking of the ground, drums are free to rock and slide either individually or in groups. Today, due to damage, destruction, and restoration purposes, many of the ancient columns are free standing, which makes them more vulnerable to the earthquake load (Komodromos et al 2008).

Over the last decades, the dynamic performance of ancient freestanding columns has received increasing scientific attention. Mainly, this was to understand their seismic performance and better select conservation and rehabilitation techniques for their survival during strong earthquakes. Housner (1963) was the first to investigate analytically the behaviour of rigid, single degree of freedom blocks subjected to horizontal excitations. From his formulations, it is possible to estimate the minimum horizontal acceleration at the base to cause overturning of the rigid body. Pioneering research activities on column mechanisms and seismic response were carried out by Milne (1881) and Omori (1900, 1902), respectively; whereas in the last four decades, many researchers studied the rocking response of rigid blocks analytically, numerically, and experimentally (Aslam et al. 1980; Yim et al. 1980; Ishiyama 1982; Psycharis and Jennings 1983; Spanos and Koh 1984; Sinopoli 1987; Tso and Wong 1989; Sinopoli 1991; Augusti and Sinopoli 1992; Shenton 1996; Andreaus and Casini 1998; Baratta et al. 2006; Lenci and Rega 2006; Drosos and Anastasopoulos 2014). Several further contributions have also considered columns composed of a small number of vertically aligned rigid blocks (Psycharis 1990; Sinopoli 1991; Augusti and Sinopoli 1992; Spanos et al. 2001; Mitsopoulou et al. 1998).

In addition over the last three decades, advanced computational methods were used to solve numerical procedures for obtaining the dynamic performance of multi-drum columns subjected to strong seismic excitations. Finite Element Modelling (FEM) is the computational tool used for most structural engineering applications. Pitilakis et al. (2017) investigated the three dimensional response of free standing columns using the finite element code ABAQUS. The FEM model has been validated with a series of shaking table tests and of a scaled multi-drum model of a column and good agreement obtained. Additional analyses conducted to investigate the seismic response of free-standing monolithic columns against those of multi-drum ones subjected to various base excitations. In addition, Lignola et al. (2014) used models based on FEM to investigate the seismic behaviour of the colonnade of the Forum in Pompeii, Italy. Both linear static and nonlinear static analyses were performed. What is more, Papadopoulos and Vintzileou (2014) used the software ABAQUS based on FEM to investigate the seismic behaviour of five multi-drum ancient columns with sufficient accuracy. The suitability of the FEM model has been validated against a series of shaking table test results from two experimental programmes on rocking of rigid bodies. Considering the sensitivity of the rocking response of dry-stone structures, the overall agreement between experimental data and numerical results was satisfactory. The finite element software was able to reproduce the key features (the frequency content, the maximum displacement and the residual slippage) of the experimentally observed dynamic response of various stone-blocks assemblies (including a multi-drum marble column).

An alternative to the available FEM is the Discrete (or distinct) Element Method (DEM). DEM was developed by Cundall for evaluating the stability of jointed or fractured rocks (Cundall 1971; Cundall and Hart 1992). The key features of the method are: a) blocks can be represented as rigid or deformable; b) large displacement and rotation of blocks is allowed; and c) new contacts are automatically detected as the simulation proceeds (Cundall and Hart 1992; Itasca 2004). In addition, large displacements and rotations between blocks, including sliding between blocks, the opening of the cracks and even the complete detachment of the blocks, and automatic detection of new contacts as the calculations proceed are allowed (Itasca 2004). DEM is particularly suitable for the analysis of blocky structures such as dry joint and low bond strength masonry where deformation results from the relative motion between the blocks (Lemos 2007). In addition, it has been shown that DEM is a very efficient approach to analyse the mechanical behaviour and geometric nonlinearity of ancient monuments, including classical columns and colonnades (Psycharis et al. 2000; Psycharis et al. 2003; Papantonopoulos et al. 2002; Pulatsu et al 2017; Papaloizou and Komodromos 2009; Sarhosis et al. 2016a; Sarhosis et al. 2016b; Pappas et al. 2016).

Winkler et al. (1995) were probably the first authors that adopted the DEM for applying harmonic excitations to columns made by up to three rigid blocks. The two-dimensional (2D) discrete element software UDEC was used by Psycharis et al. (2000) to investigate the in-plane seismic response of two multi-drum columns and identify their stability under earthquake excitations. They found that ground motions with large dominant periods are more threatening to multi-drum columns than short-period ones. Furthermore, the behaviour of multi-drum columns subjected to imperfections was investigated. Results showed that geometric imperfections including tilt or reduced contact area reduce significantly the stability of the system.

The efficiency of using DEM for predicting the seismic response of multi-drum columns was investigated by Papantonopoulos et al. (2002). The computational model developed was compared against large-scale experimental tests carried out by Mouzakis et al. (2002). The results indicated that DEM can capture quite well the main features of the response of the column. Later, Psycharis et al. (2003) and Konstantinidis and Makris (2005) investigated numerically the seismic behaviour of retrofitted multi-drum columns containing metallic shear links between them. Results showed that retrofitting with the use of stiff metallic shear links between drums results in a controlled rocking response of the column. Also, although stiff metallic shear links reduce the relative displacements between blocks, their presence could have detrimental effects to the stability of the structure.

The Universal Distinct Element Code (UDEC), based on DEM, has been used by Pena et al. (2007) to simulate the behaviour of single rocking blocks tested in the laboratory and it was found to be capable of reproducing the experimental behaviour. Furthermore, Dimitri (2009) and co-workers (2011) adopted UDEC for a parametric study of the dynamic behaviour of multi-drum columns and arches on buttresses, with particular attention to the columns of the Temple of Apollo at Bassae.

To investigate the seismic behaviour of monolithic and multi-drum classical columns and colonnades, an in-house specialised software based DEM was developed by Komodromos et al. (2008). A series of case studies was undertaken by investigating different in size and dimensions columns subjected to earthquake load. Papaloizou and Komodromos (2009) studied the influence of the frequency content and amplitude of the ground motions on the seismic response of columns and colonnades with epistyles. They found that low-frequency earthquakes endanger both of them more than high-frequency earthquakes, which complies with the experimental work carried out by Drosos and Anastasopoulos (2014, 2015). In addition, they demonstrated that colonnades with epistyles are more stable than single freestanding columns. A couple of years later, Papaloizou and Komodromos (2012) investigated the seismic behaviour of colonnade systems containing two rows of columns. They found that the required acceleration to overturn such structures decreases as the predominant frequency of the earthquake decreases. In addition, the seismic reliability of multi-drum columns has been studied by Psycharis et al. (2013) with the use of synthetic ground motions obtained from a stochastic analysis using Monte-Carlo simulations.

Recently, Sarhosis et al. (2016a, b) developed both two-dimensional and three-dimensional (3D) numerical models using the DEM software UDEC and 3DEC (Itasca 2004) to investigate the static and dynamic stability of the two-storey colonnade of the Forum in Pompeii, Italy. The structure under investigation was a three span, two-series system colonnade consisting of multi-drum columns positioned one over the other. The peculiarity of the structure is that the lower level columns support a series of both solid and segmental beams forming a flat arch. From the results analyses, it was shown that for low-frequency excitations, the primary response of the colonnade is rocking; while for high-frequency excitations, the response becomes more complicated demonstrating both sliding and rocking movements. However, Stefanou et al. (2011), using a three-dimensional software based on DEM highlighted that the dynamic behaviour of multi-drum structures such as ancient colonnades does not involve only sliding and rocking, but also wobbling. Due to wobbling, the dissipation of energy is different during seismic excitation and affects stability and deformation of the structure. Therefore, 3D DEM analyses should be better adapted to the real physics of the problem. In addition, 3D models are better suited to investigate the mechanical response of the ancient columns and colonnades since their out of plane response can be studied. However, studies carried out by Psycharis et al. (2000) and Konstantinidis and Makris (2005) showed that 2D analyses could still be used to capture the overall phenomenon and various parameters that affect the dynamic response of multi-drum columns. Moreover, they highlighted that two dimensional analysis can be used more efficiently and effectively when it is necessary to perform large numbers of simulations to study the effect of various parameters and characteristics, as 2D analysis is much more time efficient and is less sensitive to the contact parameters.

Moreover, there exist recent studies with two and three drums (spondyles) columns which under certain slenderness conditions of drums may lead to closed (exact form) solutions (Kounadis and Papadopoulos 2016; Kounadis 2018). Such analytical solutions can be used for the validation of any numerical scheme. In particular, Kounadis (2018) for the first time analytically derived the highly nonlinear differential equations of the complex rocking-sliding instability of multi-rigid block assembly. The analytical formulation allowed sliding, detachment, uplifting and rotation about one edge as well as inelastic impact and reversal rotation not of one but of two and three blocks, one on top of the other. Moreover, such analytical studies (confirmed with experimental evidence) have revealed the superiority of rocking stability behaviour of multi-drum columns, compared to "equivalent" ones (see discussion by Gazetas 2018).

The aim of this paper is to investigate the dynamic response of ancient multi-drum and monolithic columns subjected to horizontal and combined horizontal and vertical harmonic excitations. Using a software based on the Discrete Element Method (DEM) of analysis in plane state, a typical ancient multi-drum and an equivalent in dimensions monolithic columns subjected to harmonic excitations were modelled to identify the main factors affecting their stability. As previously stated, the 2D model allowed to perform a large number of simulations by applying different magnitudes of acceleration amplitude and different values of input frequency, in order to investigate their role in the collapse/deformation mechanism. Also, the influence of material parameters such as the friction angle at the interfaces between the drums was examined. Harmonic excitations are considered because, as well known, they give clear information on the effect of the frequency content of an earthquake to the possibility of collapse, whereas many seismic records contain a predominant harmonic component. Analysis results showed that harmonic excitation frequency or period plays an important role on the dynamic response of the columns.

With respect to existing contributions dedicated to the harmonic response of monolithic and multi-drum columns, this work considers the combined effect of horizontal and vertical harmonic excitations on column collapse and maximum displacements, showing that collapse conditions and displacements increase significantly with respect to the simpler case of horizontal excitation. Furthermore, this work focuses also on the serviceability state by evaluating the influence of excitations parameters on maximum horizontal displacements and drifts of the columns.

Finally, five different closely spaced multi-drum columns subjected to horizontal harmonic excitations are studied, in order to evaluate both collapse mechanisms of single columns and effects of collapsing blocks on adjacent columns.

2. Overview of UDEC for modelling multi-drum ancient columns

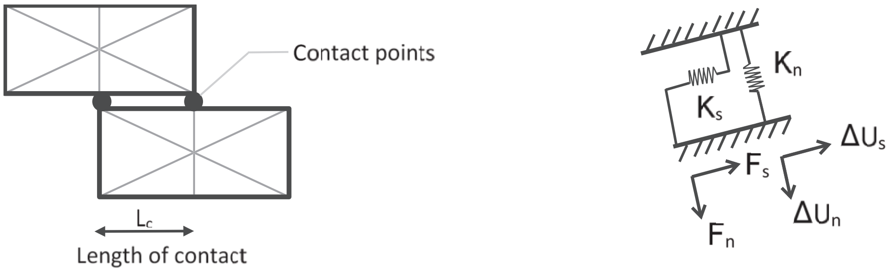
Within DEM, multi-drum ancient columns could be represented as an assembly of rigid or deformable blocks and could take any arbitrary shape. Joints are viewed as points at the surfaces where mechanical interaction between blocks takes place, governed by appropriate constitutive laws. The motion of the blocks is simulated throughout a series of small but finite time-steps, numerically integrating the Newtonian equations of motion.

2.1 Representation of the blocks

In UDEC, blocks can take any arbitrary shape and can behave in a perfectly rigid way or as deformable blocks. In the present study, the mechanical behaviour of multi-drum columns modelled as an assembly of deformable blocks to investigate the distribution of stresses inside them. For this purpose, each block is discretized into an internal mesh of triangular zones (or elements), within which a uniform strain state is assumed. As the Young's modulus of the drum materials is typically high, the internal deformations are small, and most of the column movements are due to the relative displacements between drums.

2.2 Generation of contacts

Blocks interact together by means of contact points. The contact detection algorithm defines the plane along which sliding can occur. This unit normal should change direction in a continuous fashion as the two blocks move relative to each another. In UDEC, this is achieved by assuming that the sharp vertices are rounded for contact purposes, so that the normal is well defined for both vertex-vertex and vertex-face interactions.



(a) Block to block contact

(b) Interactions at contact

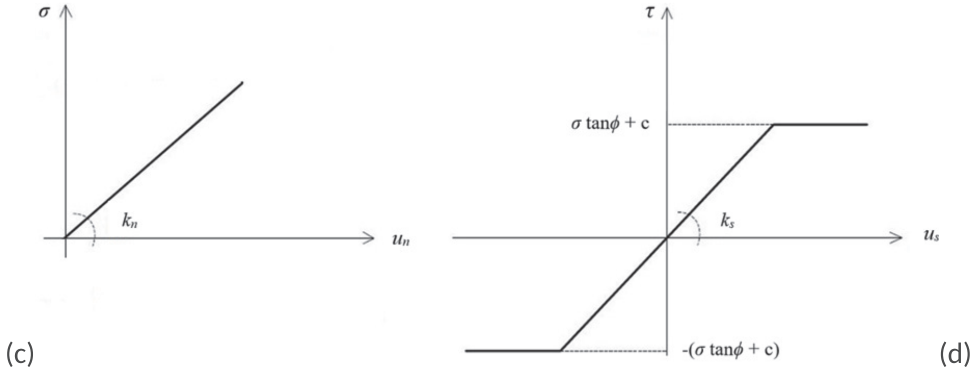


Figure 1. (a) and (b): Face-to-face contact type and corresponding sub-contacts where springs are assigned in both orthogonal directions. (c) and (d): Constitutive law describing the behaviour of joints under (c) normal and (d) shear loads

Contact exists if the overlap is positive, or equivalently, if the gap is negative between the two blocks. Sub-contacts (or contact points) are created with the help of the nodes being located on the block face (Fig. **1** a, b).

2.3 Constitutive models for contacts

The mechanical behaviour of contacts in UDEC is modelled with the help of contact stiffness defined in the normal and shear directions, relating sub-contact stresses with relative displacements characterizing the sub-contact. In the elastic range (when contact sliding and separation does not occur), the behaviour is governed by the joint normal and shear stiffnesses (k_n and k_s):

$$\Delta F^n = -\ k_n \cdot \Delta U^n \cdot A_c$$

$$\Delta F^s = -\ k_s \cdot \Delta U^s \cdot A_c$$

Where ΔF^n,ΔF^s the normal and the shear force increment, resultant for the sub-contact; k_n, k_s the joint normal and the joint shear stiffness, ΔU^n,ΔU^s the normal and the shear displacement increments belonging to the sub-contact, A_c the sub-contact area. The maximum shear force allowed is given by:

$$F_{max}^{s} = F^n \cdot \tan{(\varphi)}$$

where φ is the angle of friction. The constitutive law describing the behaviour of joints under normal and shear loads is shown in Fig. **1** c, d.

2.4 Mechanical damping

UDEC employs an explicit time stepping algorithm for the solution of both static and dynamic problems. For static analysis, damping applied in UDEC to decrease oscillations originating from the time integration technique and to facilitate a force equilibrium state as quickly as possible. Two forms of damping can be applied: a) adaptive global damping; and b) local damping. The **adaptive global damping** applies viscous damping forces, but the viscosity constant is continuously adjusted in such a way that the power dissipated by damping is a given proportion of the rate of change of kinetic energy in the system. For the local damping, different damping forces are applied on every degree of freedom. Every component is proportional to the magnitude of the unbalanced force or moment. For dynamic analysis, **Rayleigh damping** is available, consisting of the mass-proportional and stiffness-proportional components.

3. Development of the computational model

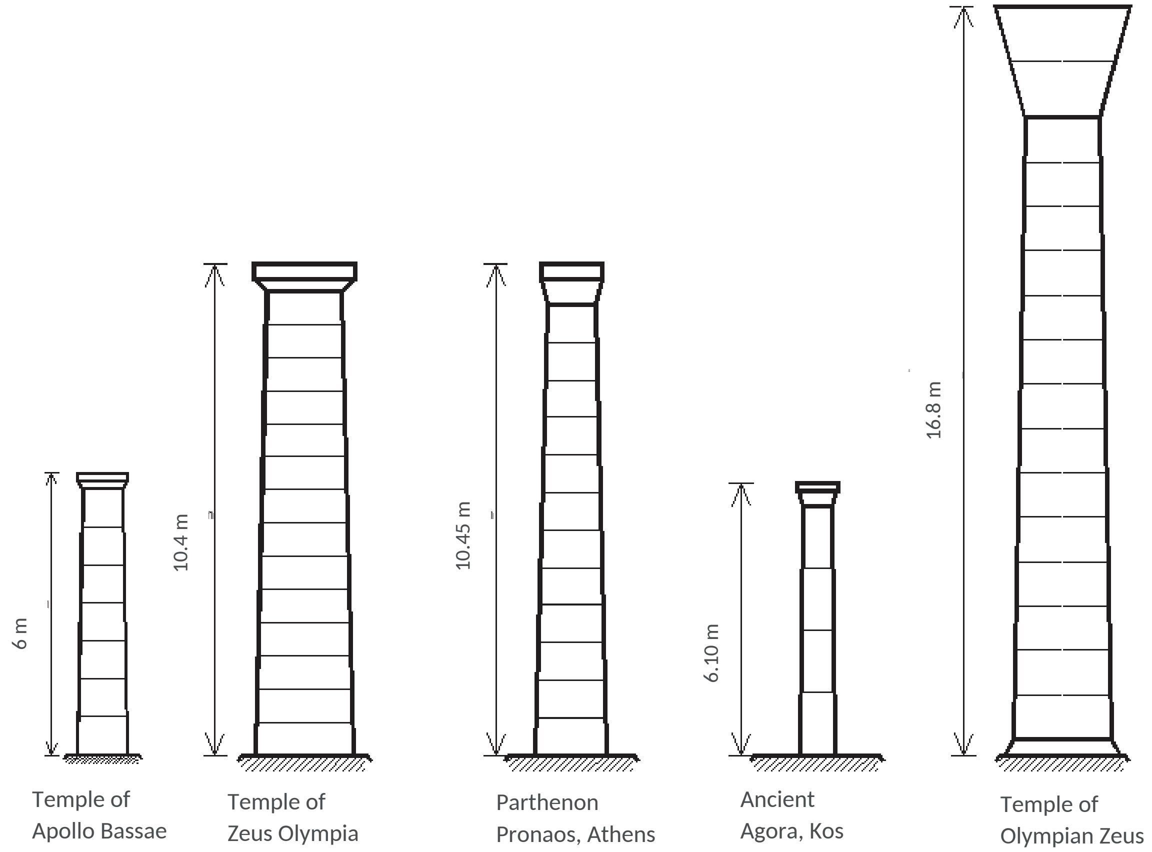
An investigation into the different geometrical properties of different columns in the Mediterranean region undertaken. Examples of different in size ancient multi-drum columns shown in Table 1. From Table 1, ancient columns vary from 3.8 m (e.g. for the column of the Forum in Pompeii, Italy) to 16.81 m (e.g. for the temple of the Olympian Zeus, Greece). In addition, ancient columns do not only vary in size but also in the number of drums (see Figure 2a). For example, the ancient column of the Forum in Pompeii has only three drums while the ancient column of the temple of the Olympian Zeus has twelve drums.

Table 1. Dimensions of ancient columns located in Italy and Greece

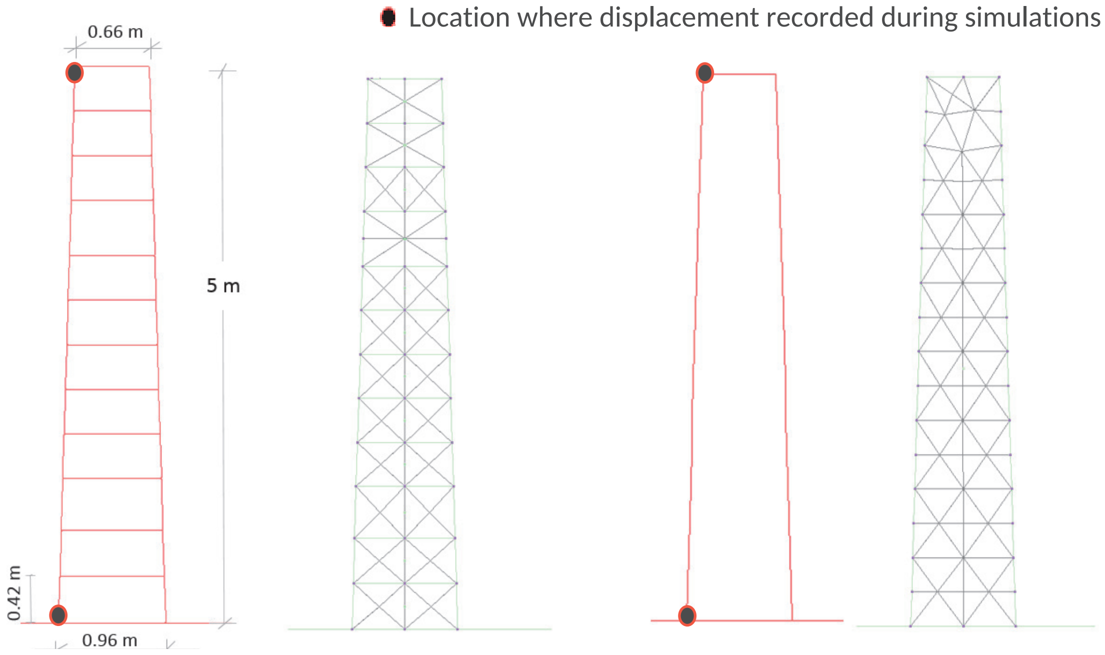
	Height (m)	Width at the base (m)	Width at the top (m)	Number of drums	Ratio of width at the base over width at the top	Ratio of height over the width of the base
	H	d_{foot}	d_{head}	n_d	$\omega = \frac{d_{foot}}{d_{head}}$	$\rho = \frac{H}{d_{foot}}$
Temple of Apollo Bassae	5.95	1.12	0.90	7	1.24	5.31
Temple of Zeus Olympia	10.44	2.22	1.70	14	1.30	4.70
Parthenon pronaos Athens	10.43	1.65	1.25	12	1.32	6.32
Ancient Agora Kos	6.10	0.78	0.64	4	1.21	7.82
Temple of Olympian Zeus	16.81	2.22	1.70	14	1.30	7.57
Poseidon Sounio	6.15	1.02	-	-	-	6.02
Junio Lacinia Agrigento	6.32	1.38	-	-	-	4.57
Forum of Pompeii, Italy	3.8	0.60	0.60	4	1.00	6.33

3.1 Geometry of the ancient columns under investigation

Geometric models created in UDEC to represent a typical multi-drum column and an equivalent in size monolithic column (Fig. **2** b, c). Having considered the variation in geometrical properties in ancient columns, In this study, a multi-drum column with height equal to 5 m consisting of 12 individual drums placed on top of each other. The ration of the width at the base to the width at the top taken as 1.45 and the ratio of the height over the width of the base was 5.21. Each drum of the ancient column was simulated using a deformable block separated by zero thickness interfaces at each joint. Blocks were internally discretized into finite-difference zone element and assumed to behave in a linear elastic manner. Material properties for the characterization of the blocks are: mass density, bulk and shear modulus. In practice, the stresses in the stone blocks would be well below their strength limit and so no significant deformation was expected to occur in them (Sarhosis et al. 2016). The zero thickness interfaces between each block modelled using UDEC's elastic perfectly plastic Coulomb criterion; defined by the elastic normal (JKn) and shear stiffness (JKs), as well as the joint angle of friction (Jfric).



(a) Monolithic column



(b) Multi-drum column

(c) Monolithic column

Figure 2. (a) Geometry of existing multi-drum columns. Geometry (b) and mesh generation (c) of a multi-drum ancient column and an equivalent in size monolithic column.

3.2 Material properties

The material parameters used for the development of the computational model shown in Tables 1 and 2. Since the multi-drum columns are dry-stacked, the joint tensile strength (J_{ten}), joint cohesive strength (J_{coh}) and the joint dilation angle (J_{dil}) were assumed to be zero. Only frictional sliding allowed to occur between adjacent drums. Values for the material parameters of the column obtained from Sarhosis et al. (2016). For all columns, the joint friction angle kept constant and equal to 30 degrees (or friction coefficient of 0.57).

Table 1 Properties of the blocks

Unit Weight p [kg/m^3]	Bulk Modulus K [Pa]	Shear Modulus G [Pa]
1,608	2.66×10^{10}	1.60×10^{10}

Table 2 Properties of the joint interfaces

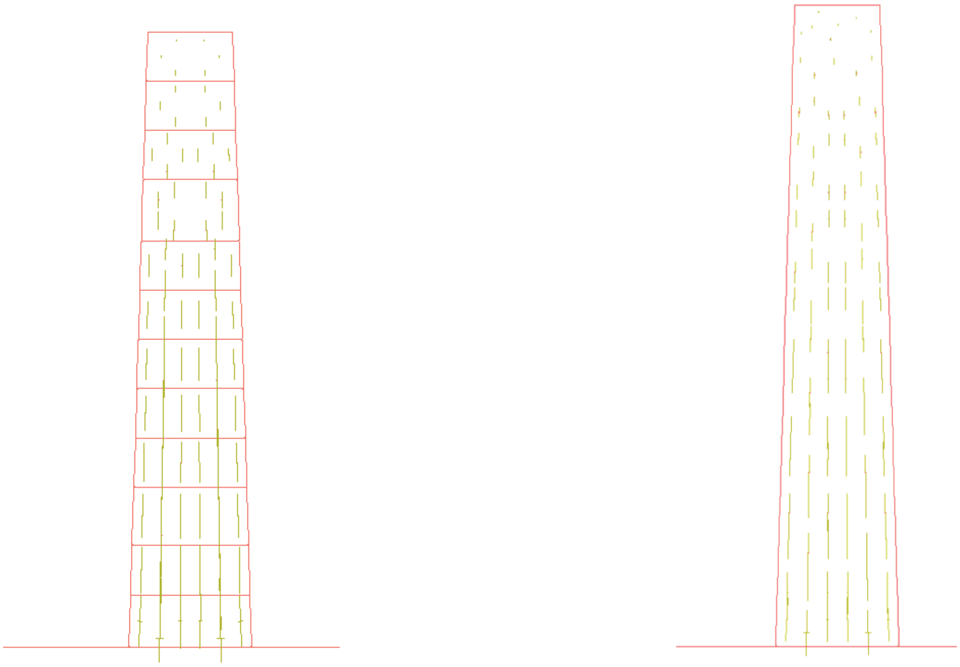
Normal Stiffness JK_n [Pa/m]	Shear Stiffness JK_s [Pa/m]	Joint friction angle J_{fric} [degrees]
5×10^{10}	2.5×10^{10}	30°

3.3 Damping

During the dynamic analysis, no viscous damping was assumed. Dissipation derived due to the presence of frictional sliding between adjacent drums. This conservative assumption is often used in stone column analysis, as damping has limited influence on the strong motion segment that causes the failure mechanisms (Psycharis et al. 2000). The analysis of shaking table tests of a drum column model also recommended low values of viscous damping to be used in the numerical models (Papantonopoulos et al. 2002). Also, for the static analysis stage, i.e. after the 10 sec of earthquake excitation, adaptive viscous damping was used, taking the default ratio of the damping and the rate of change of nodal kinetic energy of 0.5.

3.4 Loading procedure

Initially, gravitational load was assigned to the system. Then, the system brought into equilibrium under its own weight, by stepping until the unbalanced forces are almost equal to zero. Then, the dynamic analysis is initiated, applying a horizontal sinusoidal harmonic motion at the base ground in the horizontal direction. In some runs, vertical motions were also applied. Horizontal displacements at the upper left corner at the top of the column were recorded. The principal stresses for a freestanding multi-drum and monolithic columns under equilibrium are shown in Fig. **3**. As can be expected, the maximum principal stresses in tension (red color) is the same in both columns and are almost zero. In addition, compressive stresses (green color) are also very small (i.e. 8.47×10^{-4} for the multi-drum column) and are due to the distribution of the self-weight in the column.



(a) Multi-drum column (b) monolithic column

Figure 3. Principal stresses of (a) multi-drum (max compression 67.4 kN/m^2); and (b) monolithic ancient column (max compression 67.1 kN/m^2) (Stresses are in N/m^2; compressive stresses are negative and shown with a green color while tensile stresses are positive and shown with a red color)

4. Numerical results - response to harmonic excitation

A parametric investigation of the response of the columns subjected to harmonic excitations in horizontal and both in horizontal and vertical direction is undertaken. The aim of the study is to understand the influence of the excitation frequency and acceleration magnitude on the displacements and collapse mechanisms of the columns. The frequency f is assumed to vary from 1 to 4 Hz and the amplitude of the base acceleration a_g is assumed to vary from 0.1 g to 0.5 g, leading to a set of 30 different dynamic analyses for each column. The behaviour of a multi-drum column is compared with that of monolithic one having equivalent dimensions (see Fig. **2** c, d). In addition, the effect of harmonic load on multi-drum column characterized by a reduced joint frictional resistance is investigated. Finally, the dynamic behaviour of five closely spaced multi-drum columns is taken into account, with particular attention to collapses due to the possible contacts between neighbouring columns. All results and discussion are presented below.

4.1 Performance of monolithic column subjected to harmonic load applied in the horizontal direction

Starting with the simpler case of the monolithic column (Fig. **2** c) subjected to harmonic horizontal excitations, Fig. **4** collects the displacement at the base/ground and the displacement at the top corner of the column against real time for several combinations of input acceleration magnitude and frequency. After defining column drift as the difference between top and base horizontal displacements over the height of the column, Fig. **5** shows the absolute drift values against real time for several harmonic excitations. The deformed configurations and collapse mechanisms at the end of each analysis are collected in Fig. **6**.

With the lower acceleration magnitude a_g = 0.1 g and varying frequency, base and top displacements turn out to be almost coincident, drifts are always less than 0.003% (first row of Figs. **4** and **5**), respectively, showing a linear elastic behaviour of the column with negligible residual displacements, which are not showed in Fig. **6**.

Considering larger acceleration magnitudes up to a_g = 0.5 g, top displacements are larger than base displacements, drift values tend to increase, in particular if excitation frequency decreases. With f equal to 4 and 2 Hz, displacements remain less than 0.2 m, drift values remain generally less than 5%, and columns are subjected to small rigid horizontal displacements, namely base shear displacements, without collapsing. Considering then $f \leq 1.33$ Hz, top displacements are larger than 0.2 m, and in some cases larger than 1 m, with drift values larger than 100%, corresponding to the collapse of the column due to rocking and overturning.

Fig. **7** a, b resumes the results of the dynamic analyses, by showing the contour maps of the maximum absolute top displacements and maximum drift values, respectively, experienced by the column during the dynamic analyses for varying input acceleration magnitude and frequency. Furthermore, the well-known safe-unsafe domain (Spanos and Koh 1984) is superimposed to each plot with a black continuous line. The safe-unsafe boundary is estimated by recording the collapse status of each dynamic analysis and by considering the acceleration amplitude required to cause the collapse of the column. Such amplitude decreases for decreasing excitation frequency and low frequency excitations are more prominent to cause a structural collapse.

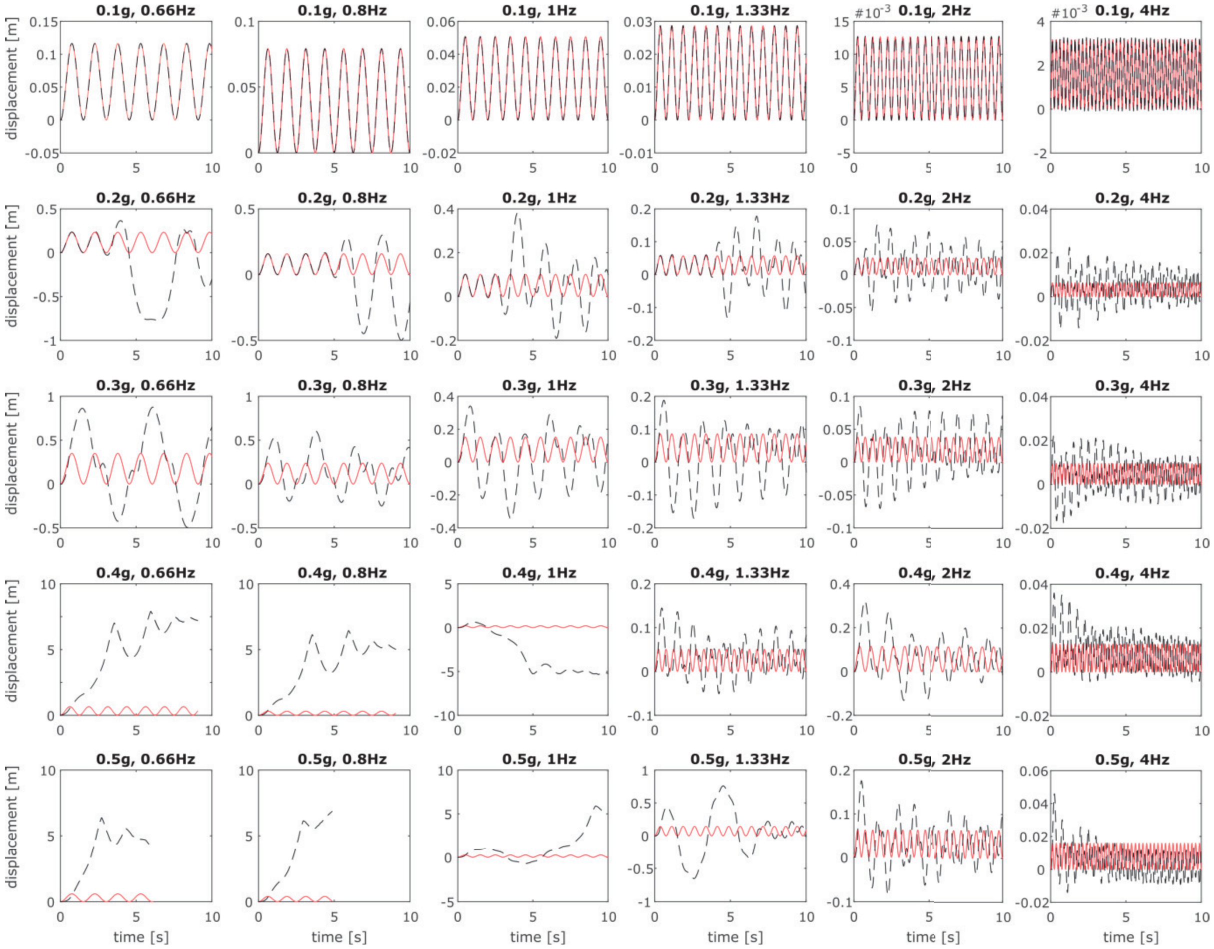


Figure 4. Monolithic column subjected to several combinations of input acceleration magnitude and frequency in horizontal direction: horizontal displacements at the base of the column (continuous lines) and at the top corner of the column (dashed lines) against time.

As can be expected, a very good correspondence between the largest displacement/drift values with the safe-unsafe domain was obtained, with extremely large maximum displacements and drift values corresponding to the unsafe domain and smaller, but not negligible, maximum displacements and drift values for the safe domain. In fact, large displacement and drift values obtained with f > 1.33 Hz and a_g > 0.3 g are not sufficient for causing a rocking collapse of the column. Such information in terms of maximum displacement turns out to be useful for defining a safety distance around the column, in order to avoid impacts and damage to adjacent objects or structural elements due to contact.

The static ultimate load multiplier for this type of column due to overturning, which may represent the case of input frequency tending to zero, can be easily determined by performing a limit analysis and applying the virtual work principle (or momentum equilibrium): $\lambda_s = b/H = 0.96/5.0 = 0.192$. Such a value is added to the contour maps by means of a black dashed line and it represents a conservative lower bound for the structural stability of the monolithic column. It is worth noting that such a simple formula was already introduced in the pioneering contribution by Milne (1889), known as West’s formula, which represents the acceleration magnitude able to initiate the rocking of a rigid block.

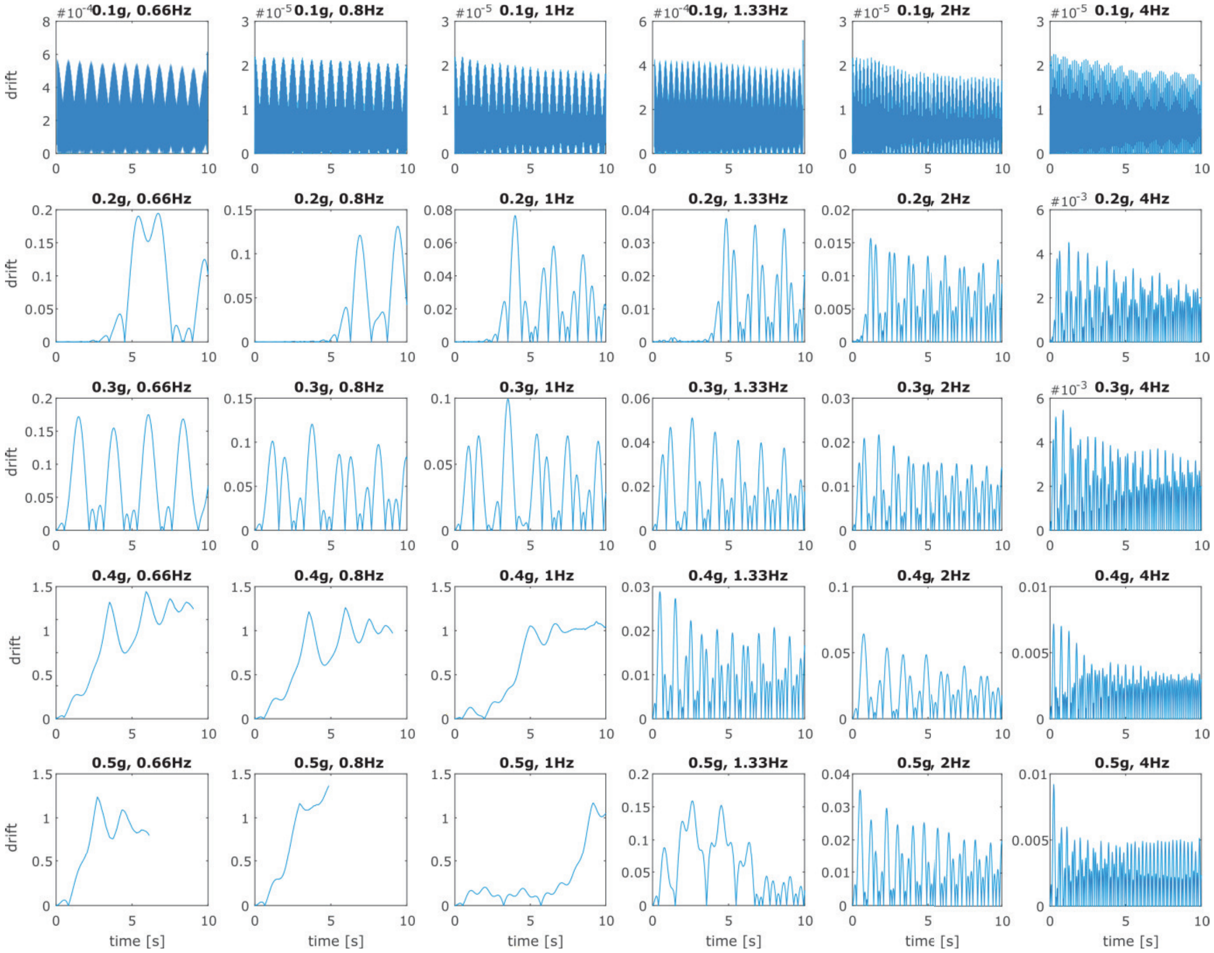


Figure 5. Monolithic column subjected to several combinations of input acceleration magnitude and frequency in horizontal direction: absolute relative horizontal displacement/drift.

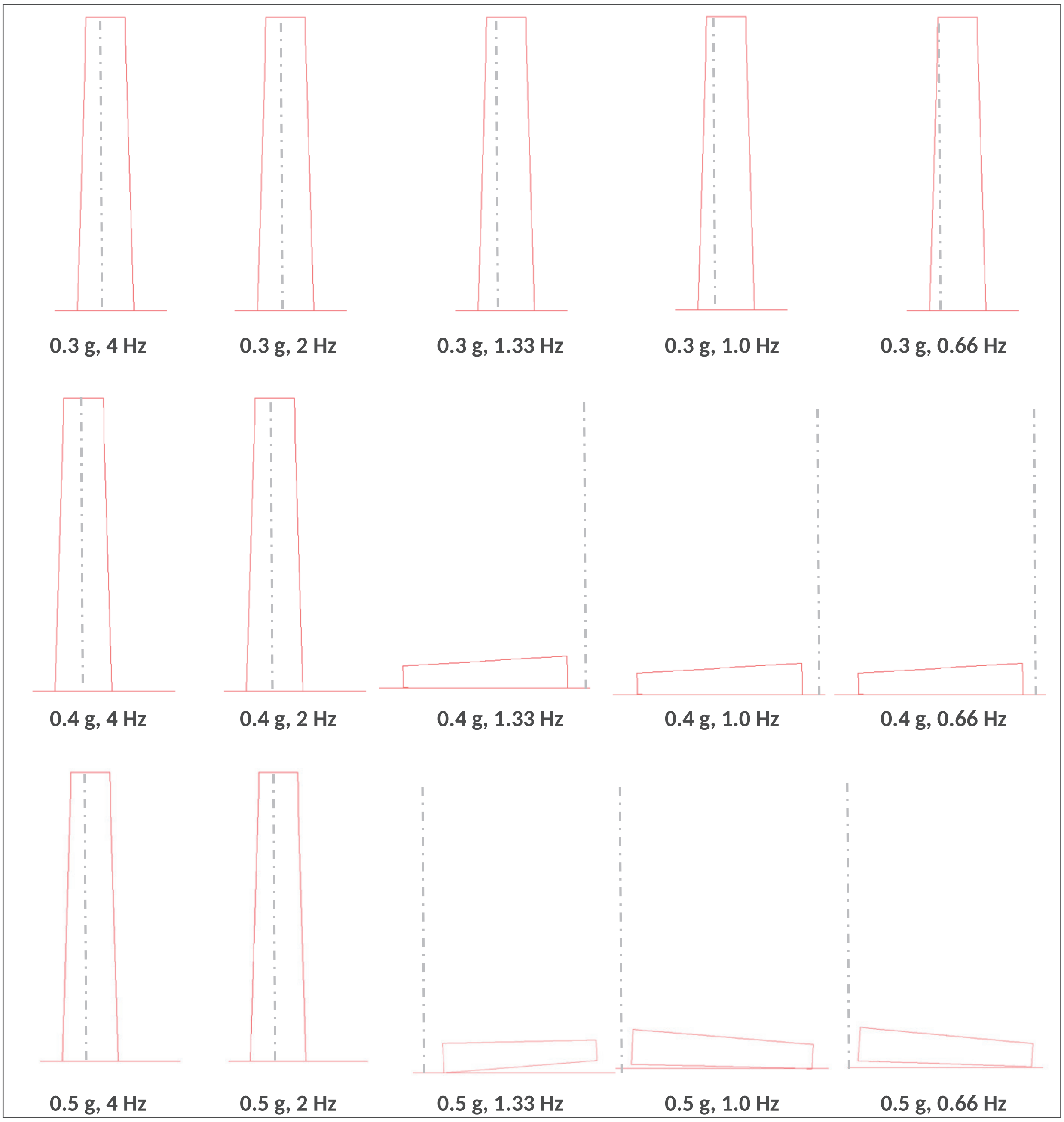


Figure 6. Monolithic column subjected to several combinations of input acceleration magnitude and frequency in horizontal direction: final deformed shape. Note: dashed line denotes the original centre line position of the drums.

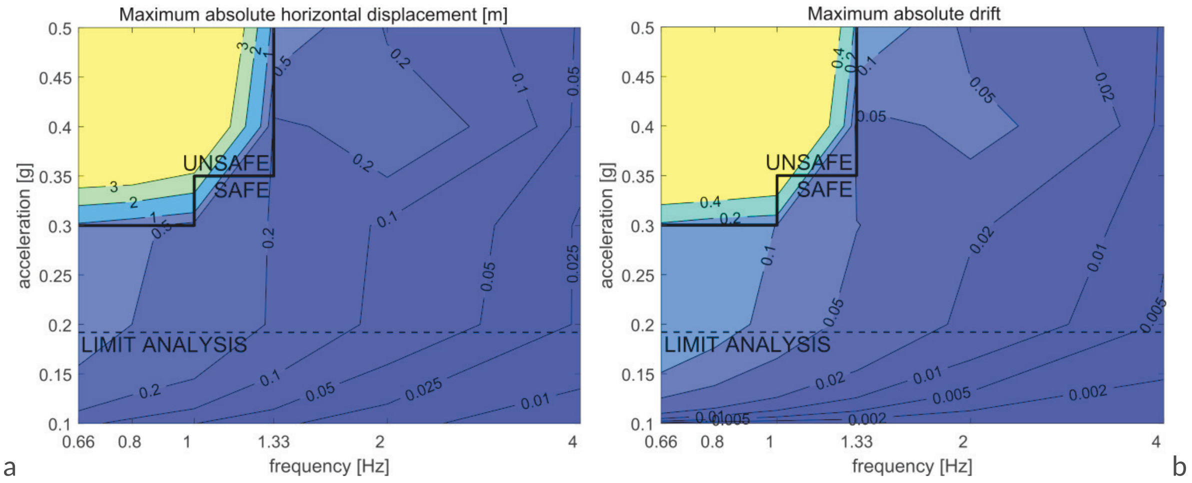


Figure 7. Monolithic column subjected to harmonic excitation in the horizontal direction: (a) maximum absolute horizontal displacement at the top; (b) maximum absolute drift.

4.2 Performance of multi-drum column subjected to harmonic load applied in the horizontal direction

The case of the multi-drum column subjected to combined horizontal and vertical harmonic excitation, using the same motions in both directions, were investigated. Although this is rather a conservative assumption, it may still be true for ancient columns located close to faults. Fig. **8** shows the horizontal displacements at the base/ground and at the top corner of the column against time for several combinations of input acceleration magnitude and frequency of excitation. Fig. **9** shows the drift values, whereas Fig. **10** shows the final deformed configurations and collapse mechanisms.

Similarly to the monolithic column case, Figs. **8** and **9** show that the period of the prescribed base motion significantly influences the mechanical behaviour and deformability of the multi-drum column. For low acceleration magnitude, namely a_g = 0.1 g, the displacement at the base/ground is practically equal to the displacement at the top of the column, due to the stiff behaviour of the structure in the linear range and thanks to the friction effect of the joints between the drums. For this reason, the corresponding deformed shapes are not represented in Fig. **10**. However, increasing the acceleration magnitude, the behaviour of the column becomes non-linear and relative displacements and rotations between the drums arise during the numerical simulations. In particular, if values of vibration frequency are larger than 1 Hz, maximum horizontal displacements at column top are always less than 0.2 m and the corresponding drift values are less than 10%, corresponding to deformed shapes characterized by the slip of horizontal joints due to their shear failure, but without column collapse. Top displacements and drift values turn out to increase up to 1 m and more than 20%, respectively, for decreasing frequency, namely $f \leq 0.8$ Hz and $a_g \geq 0.3$ g. Such displacements correspond to collapse mechanisms characterized by the rigid rotation of a column portion with respect to the lower drums.

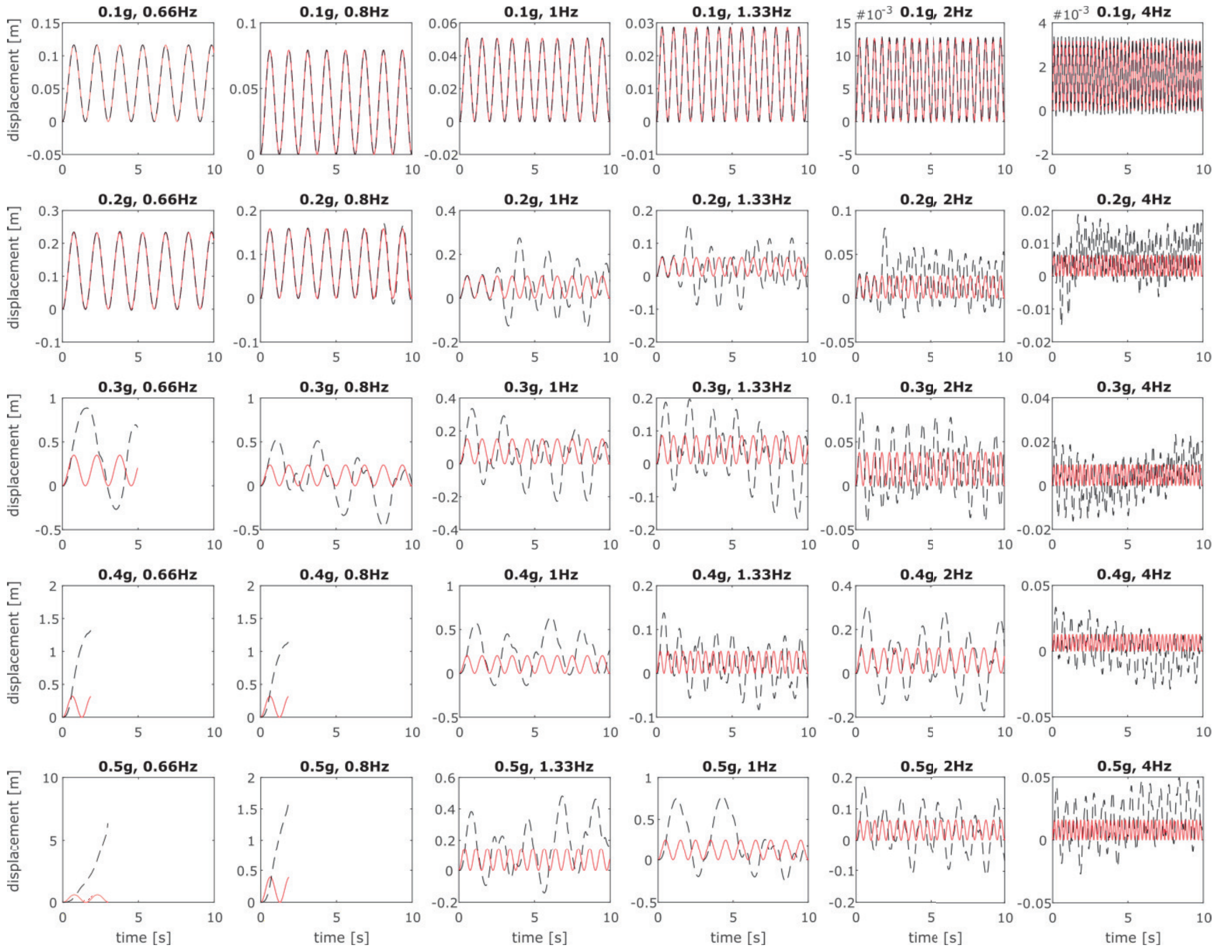


Figure 8. Multi-drum column subjected to several combinations of input acceleration magnitude and frequency in horizontal direction: horizontal displacements at the base of the column (continuous lines) and at the top corner of the column (dashed lines) against time.

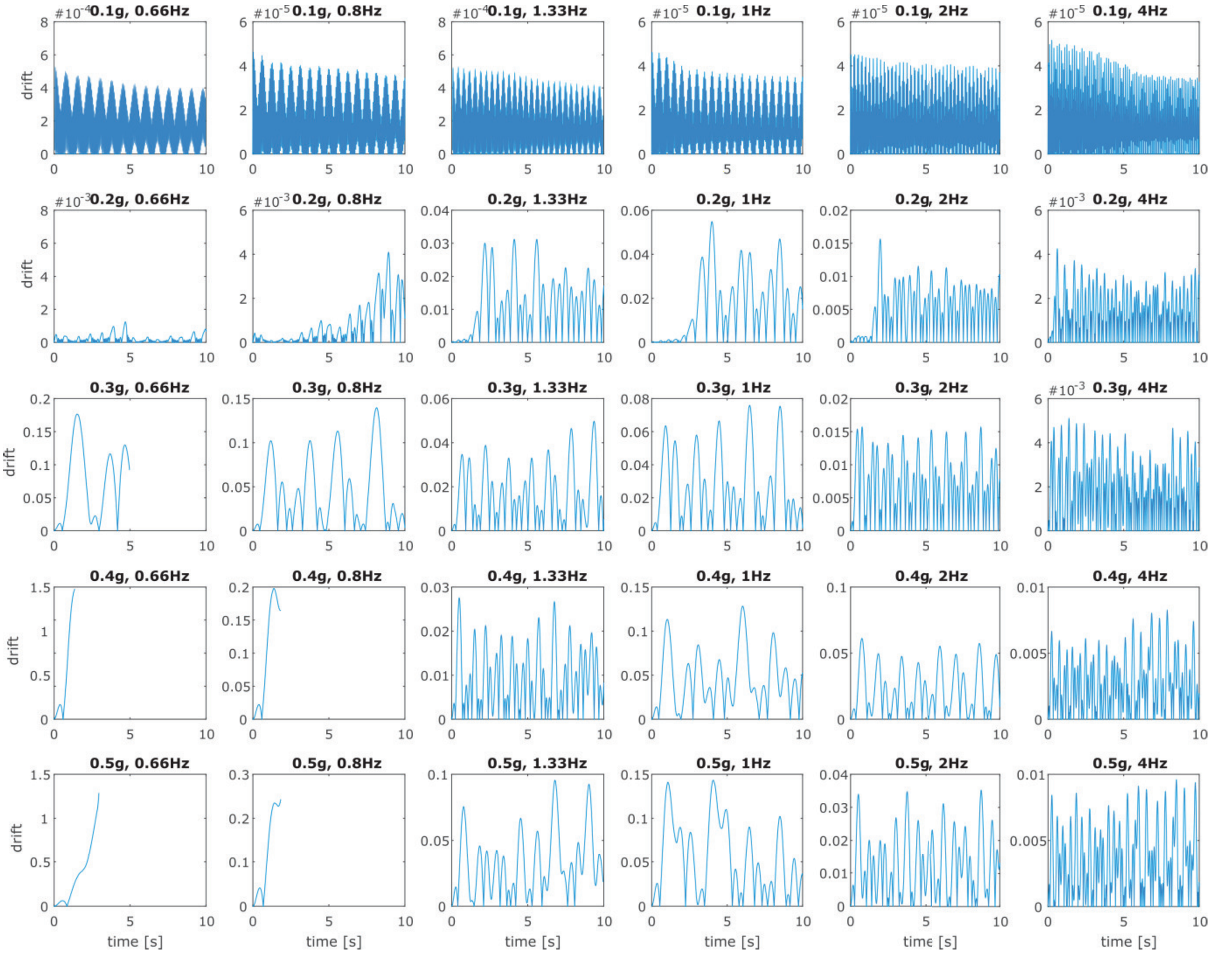


Figure 9. Multi-drum column subjected to several combinations of input acceleration magnitude and frequency in horizontal direction: absolute relative horizontal displacement/drift.

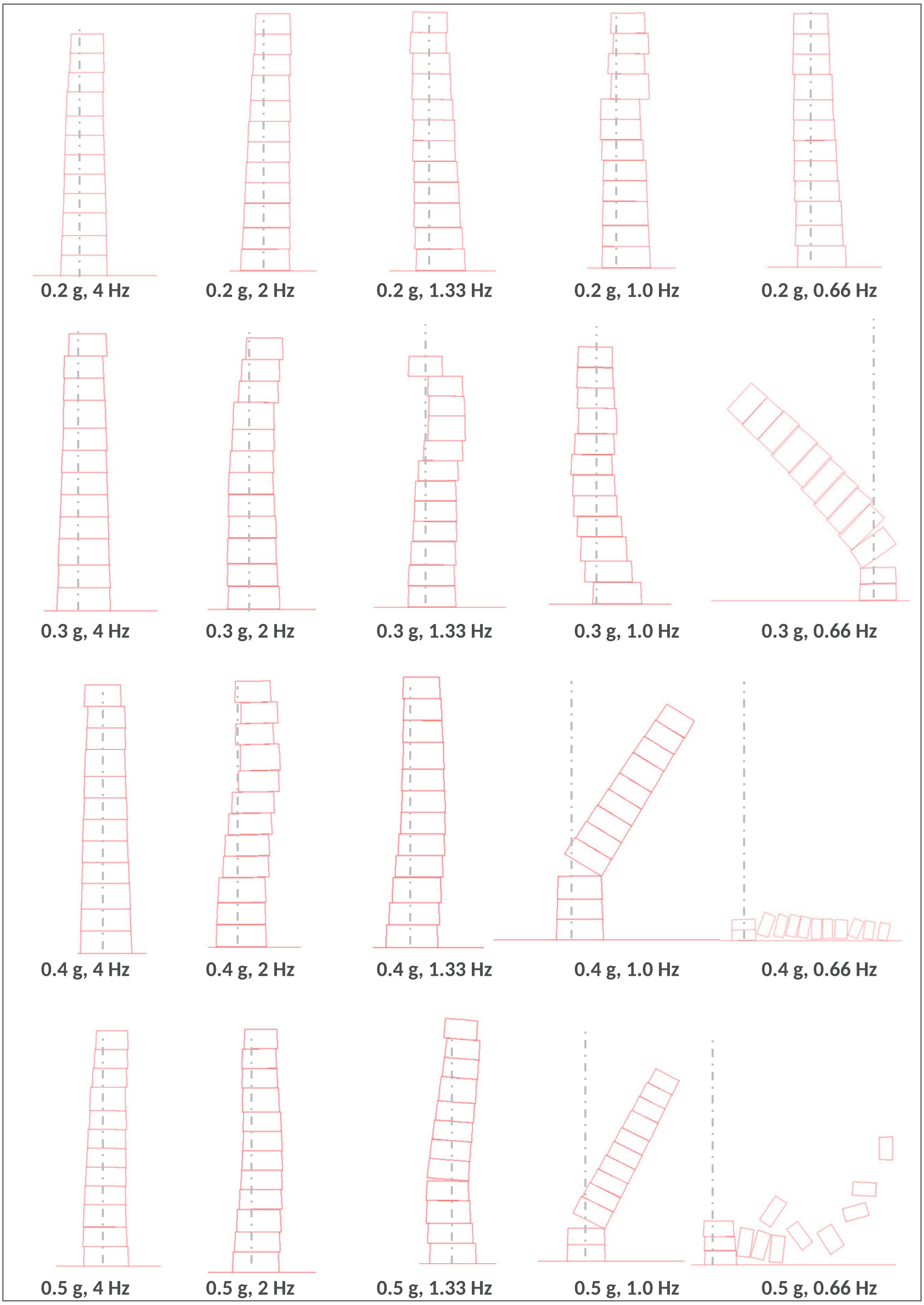


Figure 10. Multi-drum column subjected to several combinations of input acceleration magnitude and frequency in horizontal direction: final deformed shape. Note: dashed line denotes the original centre line position of the drums.

Further considerations can be done by focusing on the final deformed configurations of the multi-drum column subjected to different magnitudes of acceleration and frequency (Fig. **10**). Considering these series of analyses, it can be found that for excitation frequencies that are usually encountered in earthquakes in Mediterranean region, intact freestanding multi-drum column could withstand large amplitude harmonic excitations without collapse. For example, the freestanding column subjected to harmonic base motion of 0.25 sec period, namely f = 4 Hz, does not overturn even for a base acceleration amplitude as large as a_g = 0.4 g or 0.5 g. However, such dynamic resistance significantly reduces as the period of harmonic excitation increases (or the frequency of harmonic excitation reduces). In addition, from Fig. **10**, it is evident that the failure mode is dependent on the acceleration magnitude and excitation frequency. In fact, a column subjected to high frequency motion (e.g. f = 2 Hz) is subjected to inter-slip failure between the drums (e.g. shear failure), but it does not collapse for overturning. A similar deformed configuration can also be seen for example in a multi-drum column shown in Fig. **12**, which is located at the Propylea Athenian Acropolis, Greece. However, for low excitation frequencies (f = 0.66 Hz), a monolithic response of the column is observed soon after rocking of the base drums of the column occurs. Similar results were obtained by Psycharis et al. (2000) and Dimitri et al. (2011) by investigating the harmonic behaviour of the multi-drum column of the temple of Bassae and the experimental tests carried out by Housner (1963).

Fig. **11** resumes the results of the dynamic analyses, by showing the contour maps of the maximum absolute top displacements and maximum drift values, respectively, for varying input acceleration magnitude and excitation frequency. Similarly to the previous case, the safe-unsafe domain is superimposed to the contour plots, showing that the amplitude that causes collapse decreases for decreasing excitation frequency and low frequency excitations are more prominent to cause a structural collapse. Furthermore, similarly to the previous case, an excellent agreement between the unsafe domain and the largest maximum displacements and drift values is obtained. Considering the limit case of a static load (namely frequency tending to zero), it can be easily demonstrated by performing a series of consecutive limit analyses studies to each drum that the lowest ultimate load multiplier for a multi-drum column is coincident with that of the equivalent monolithic column (Giuffrè, 1991). Hence, λ_s = 0.192 represents again a conservative lower bound for the structural stability of the column.

It is worth noting that the information in terms of maximum top displacements experienced by the column during the harmonic excitation turns out to be useful for defining a safety distance around the column, in order to avoid impacts and damage to adjacent objects or structural elements due to contact.

By comparing monolithic and multi-drum columns subjected to the same horizontal dynamic inputs, top displacements, drifts and final deformed shapes turn out to be coincident only with a_g = 1. In this case, with the lowest value of acceleration amplitude, a multi-drum column behaves like a monolithic one. A nearly-monolithic behaviour of the multi-drum column can be found also with the analyses performed with the largest input frequency considered f = 4 Hz, independently on the applied acceleration magnitude.

Increasing a_g and decreasing f, the behaviour of the multi-drum column starts to be different than that of the monolithic one, due to the slip of horizontal joints. However, with excitation frequencies larger than 1.33 Hz, such slip does not cause large displacement at the drums and do not cause column collapse due to overturning or falling blocks. Rocking collapse mechanisms typical of monolithic columns are partially obtained with the multi-drum columns, with a_g > 0.3 g and $f \leq 1$ Hz.

The monolithic columns are generally more prone to collapse when compared to multi-drum columns, since the joints between the drums act as a dissipation device allowing deformations between them to occur. Furthermore, for the same reason, horizontal displacements and drift values experienced during the harmonic excitations by a multi-drum column are generally smaller than those of a monolithic one. However, results in terms of maximum absolute displacements and drifts for $f \geq 1.33$ Hz turn out to be coincident for both column types (Figs. **7** and **11**). Furthermore, maximum displacements and drift values corresponding to the development of unsafe conditions turn out to be almost the same for both column types.

The similarities in the response between the monolithic and multi-drum column observed could potentially denote that for low magnitude of ground acceleration and increasing input frequency, a much simpler single block analysis can be used for obtaining an approximate prediction of the dynamic performance and stability of a multi-drum model.

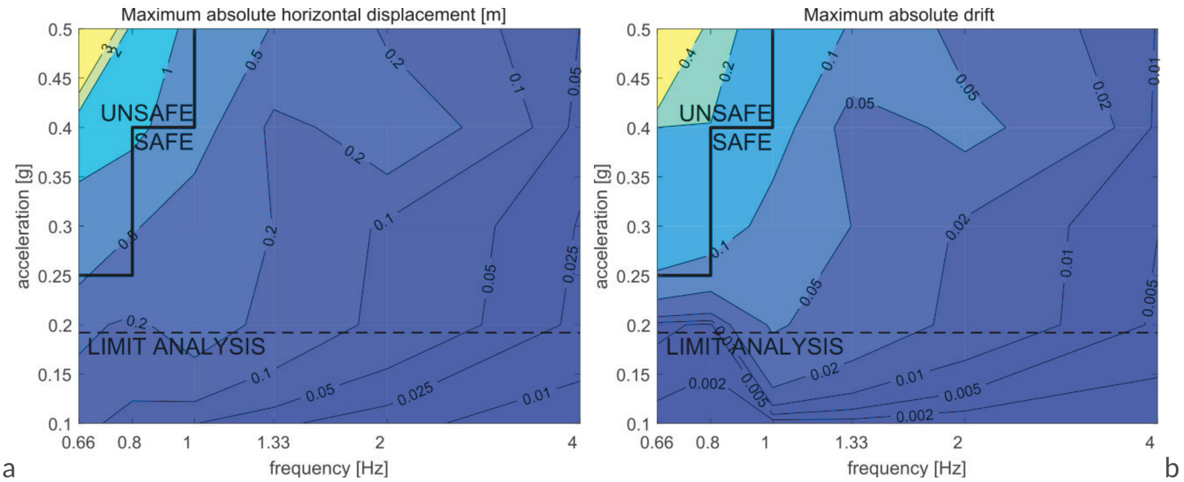


Figure 11. Multi-drum column subjected to harmonic excitation in the horizontal direction: (a) maximum absolute horizontal displacement at the top; (b) maximum absolute drift.

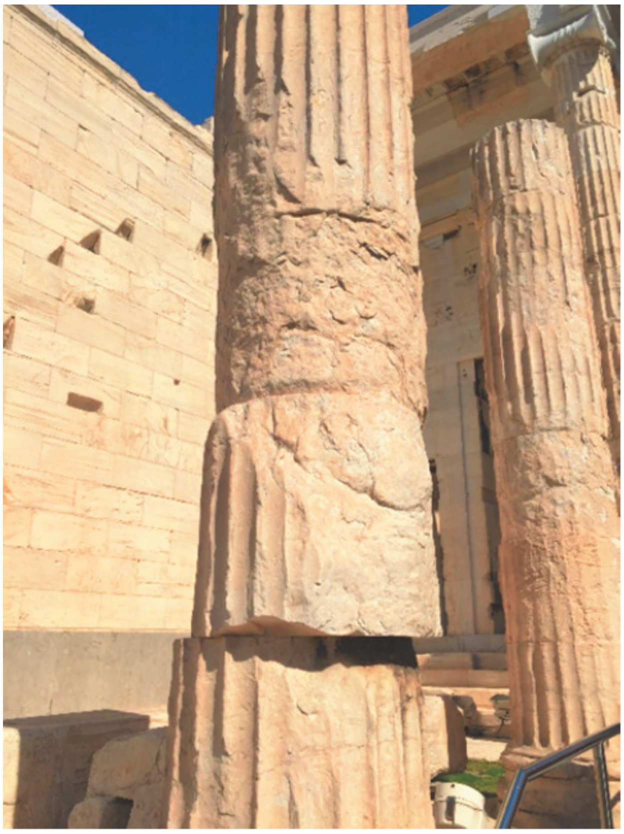


Figure 12. Shear failure between blocks in a multi-drum column located in Acropolis in Athens, Greece.

4.3 Influence of friction angle of the joints on the dynamic performance of multi-drum columns

An investigation into the influence of the joint frictional angle at the interface between drums was also undertaken. In this instance, the friction angle was lowered from 30 degrees (or 0.577 Coulomb friction coefficient) adopted previously to 20 degrees (or 0.364 Coulomb friction coefficient). Fig. **13** shows the ground displacements and displacements at the top corner of the column against time for the different combinations of input acceleration magnitude and frequency. Comparing Fig. **8** with Fig. **13**, it is evident that columns with lower friction angle can develop larger displacements. Shear slip between the joint occurs which could lead to larger displacement and potentially failure. This is evident for lower frequencies and higher ground accelerations.

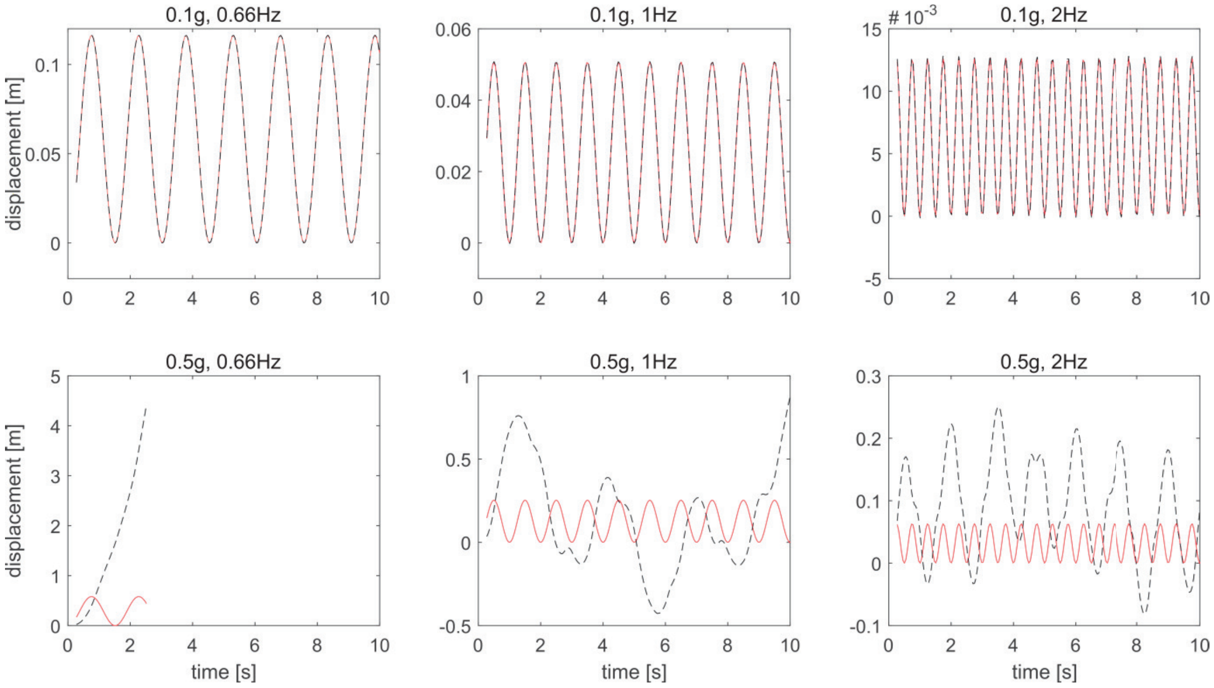


Figure 13. Multi-drum column with lower frictional resistance subjected to several combinations of input acceleration magnitude and frequency in horizontal direction: horizontal displacements at the base of the column (continuous lines) and at the top corner of the column (dashed lines) against time.

The collapse mechanism of multi-drum columns with low joint frictional resistance are presented in Fig. **14**, where the shear behaviour of the columns is evident. A low angle of friction at the interfaces of the column allow slippage between the drums and thus are more prone to failure. This is evident by comparing safe and unsafe state of the column with low and high frictional resistance; in fact, considering Fig. **11** and Fig. **15**, columns with lower joint friction are more prone to collapse than those with high friction angle.

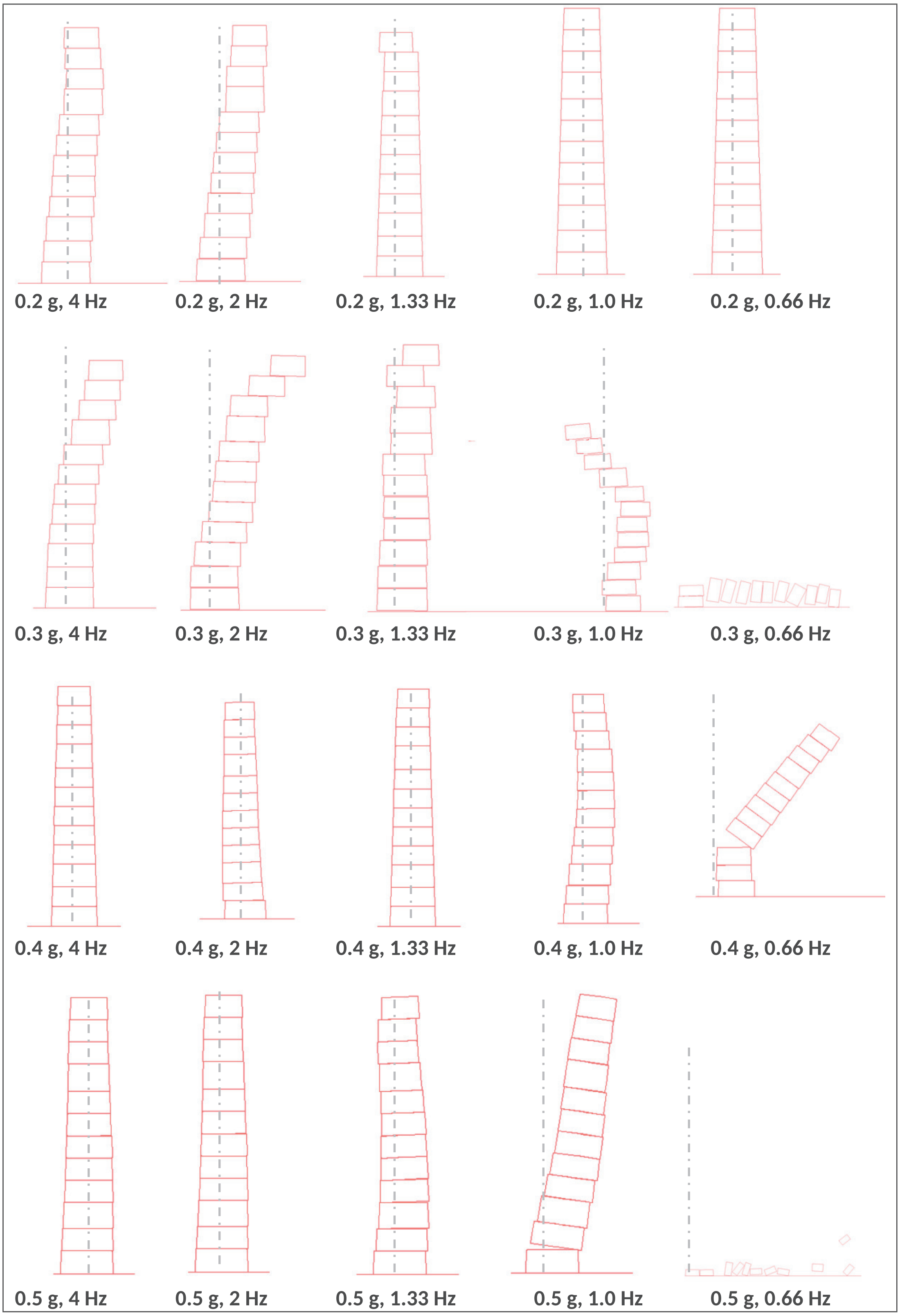


Figure 14. Multi-drum column with lower joint frictional resistance subjected to several combinations of input acceleration magnitude and frequency in horizontal and vertical direction: final deformed shape. Note: dashed line denotes the original centre line position of the drums.

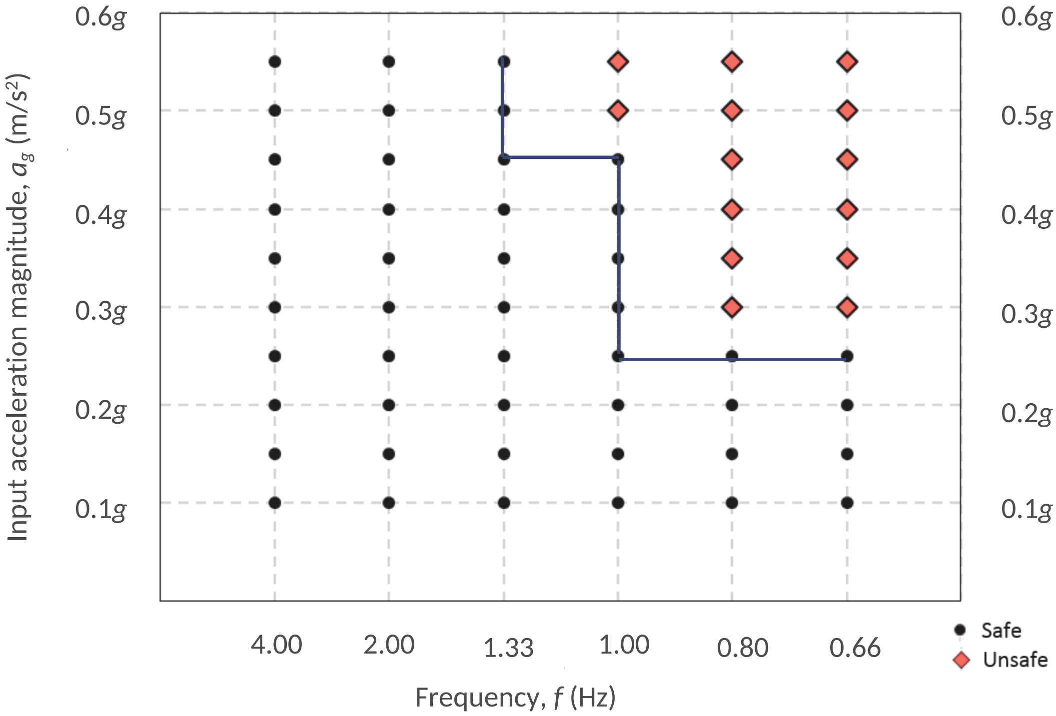


Figure 15. Safe-unsafe boundary for a freestanding multi-drum column with reduced joint frictional resistance subjected to harmonic excitation in the horizontal direction.

4.4 Performance of monolithic column subjected to harmonic load applied in the horizontal and vertical direction

An investigation into the influence of applying both horizontal and vertical harmonic excitations at the base of the multi-drum column was undertaken. In this instance, the amplitude of the horizontal and vertical acceleration assigned in both directions were the same and occurred at the same time equivalently. This type of action, characterized by both horizontal and vertical excitations, is typical of seismic events in Greece and Greek islands (Pitilakis and Roumelioti 2013).

Figure **16** shows the horizontal displacements at the base/ground and displacement at the top corner of the column against time for several combinations of input acceleration magnitude and frequency of excitation. Fig. **17** shows the drift values in horizontal direction, whereas Fig. **18** shows the final deformed configurations and collapse mechanisms.

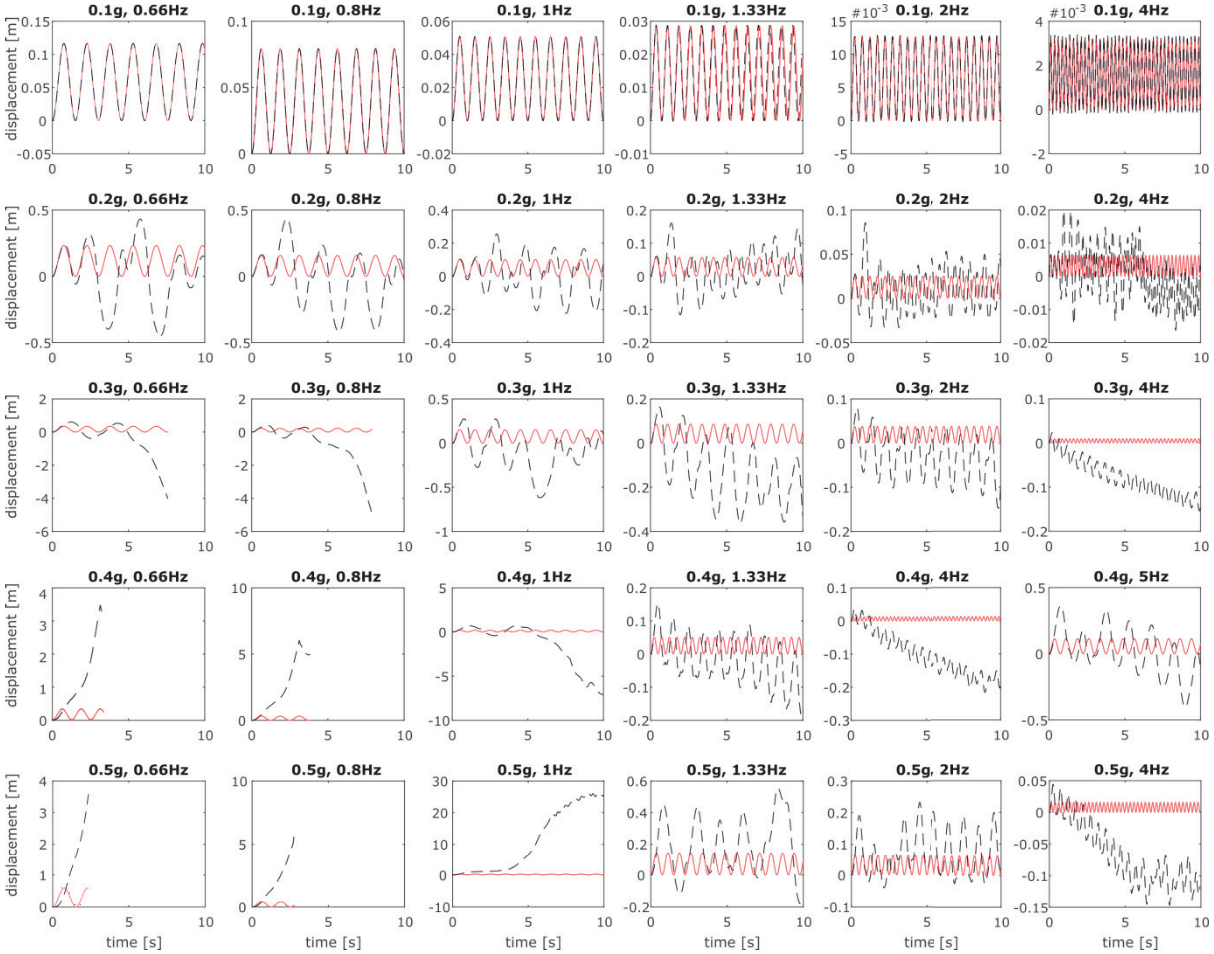


Figure 16. Multi-drum column subjected to several combinations of input acceleration magnitude and frequency in horizontal and vertical direction: horizontal displacement at the base of the column (continuous lines) and at the top corner of the column (dashed lines) against time.

From Fig. **16**, the displacements are significant for high magnitude of ground acceleration and turn out to be generally larger than those obtained with only horizontal excitation (Fig. **8**). Furthermore, in this case, larger top displacement with respect to the base and, consequently, larger drift values are obtained already with a_g = 0.2 g (second row in Figs. **16** and **17**), especially with small input frequency. Comparing Fig. **10** and Fig. **18**, it is evident that columns subjected to harmonic excitations in both horizontal and vertical directions are more prone to collapse when compared to those subjected to horizontal accelerations. The excitations with a_g = 0.1 g do no activate any relative displacement between the drums and the column behaves as a monolithic one, similarly to the previous case. Then displacements start to be significant, larger than 0.1 m, already with a_g = 0.2 g and $f \leq 1$ Hz. Furthermore, the corresponding drift values are large than 20%. However, the final configurations are not characterized by collapse mechanisms (Fig. **18**). Considering low frequency values, namely $f \leq 1$Hz and $a_g \geq 0.3$ g, displacements increase significantly leading to the collapse of the column. Fig. **18** shows the collapse mechanism observed, characterized by the overturning of almost all the drums of the column. Furthermore, in several cases, the base of the column sheared excessively and led to rocking behaviour allowing the column to collapse in a monolithic mode (a_g = 0.3 g and 0.4 g, $f \leq 1.0$ Hz).

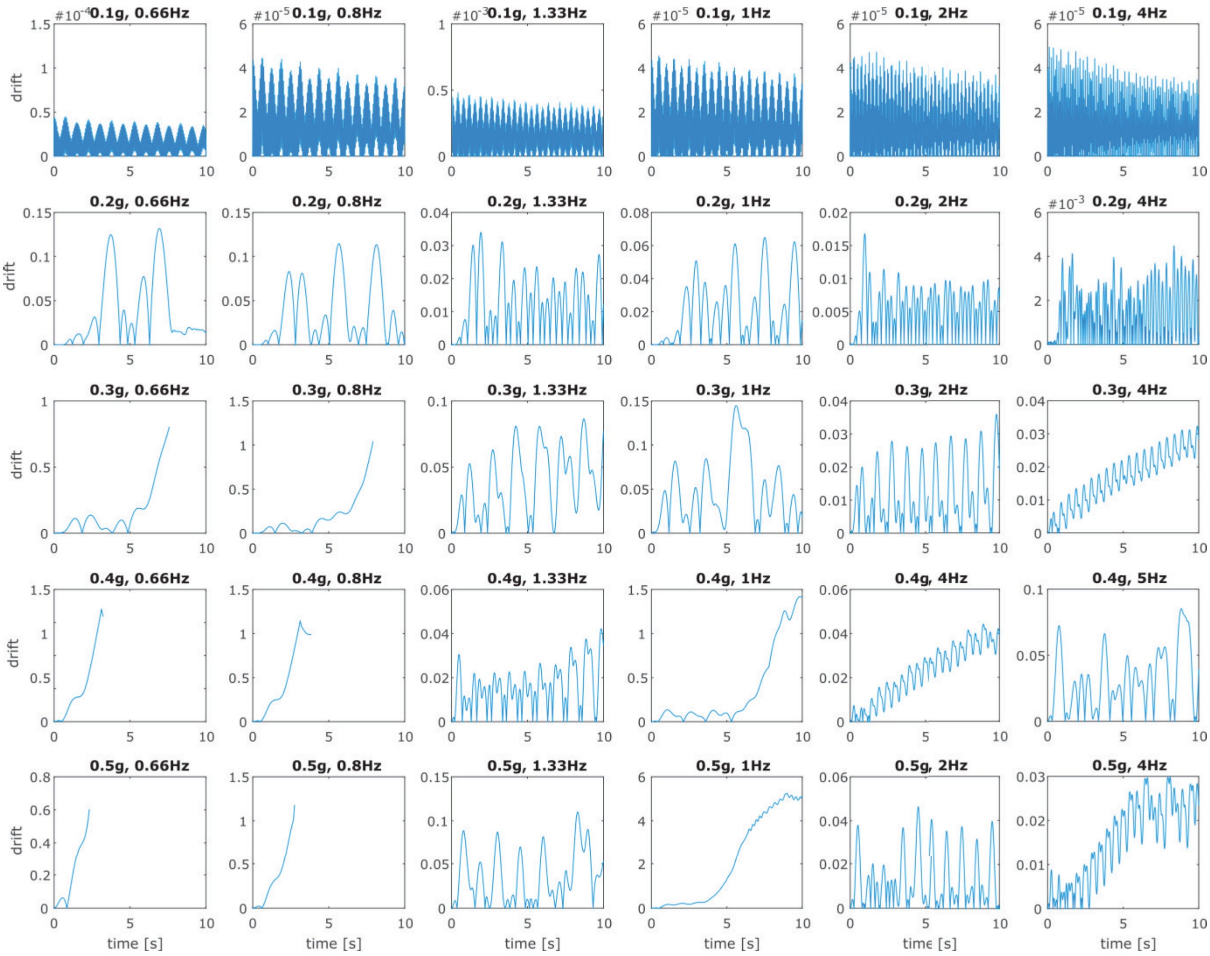


Figure 17. Multi-drum column subjected to several combinations of input acceleration magnitude and frequency in horizontal and vertical direction: absolute relative displacement/drift.

Fig. **19** a, b resumes the results of the dynamic analyses, by showing the contour maps of the maximum absolute top horizontal displacements and maximum drifts, respectively, for varying input acceleration magnitude and excitation frequency. The safe-unsafe domain is superimposed to the contour plots. Large displacements and drift values, together with collapse mechanisms, are obtained for increasing ground acceleration and decreasing frequency. In this case, larger displacements/drift values and a larger unsafe domain is obtained with respect to the multi-drum column subject to horizontal excitation only, due to the additional vertical acceleration. In particular, column collapse with low frequency can be achieved at a_g = 0.3 g; furthermore, collapse can be also achieved at frequencies up to 1.33 Hz and a_g = 0.5 g. However, due to the excitation in both directions, Fig. **19** shows that the column can be subject to very large displacements and drifts during the analysis and the maximum displacements and drift values corresponding to the safe-unsafe limit are generally larger than those typical of the multi-drum column subject to horizontal excitations (Fig. **11**).

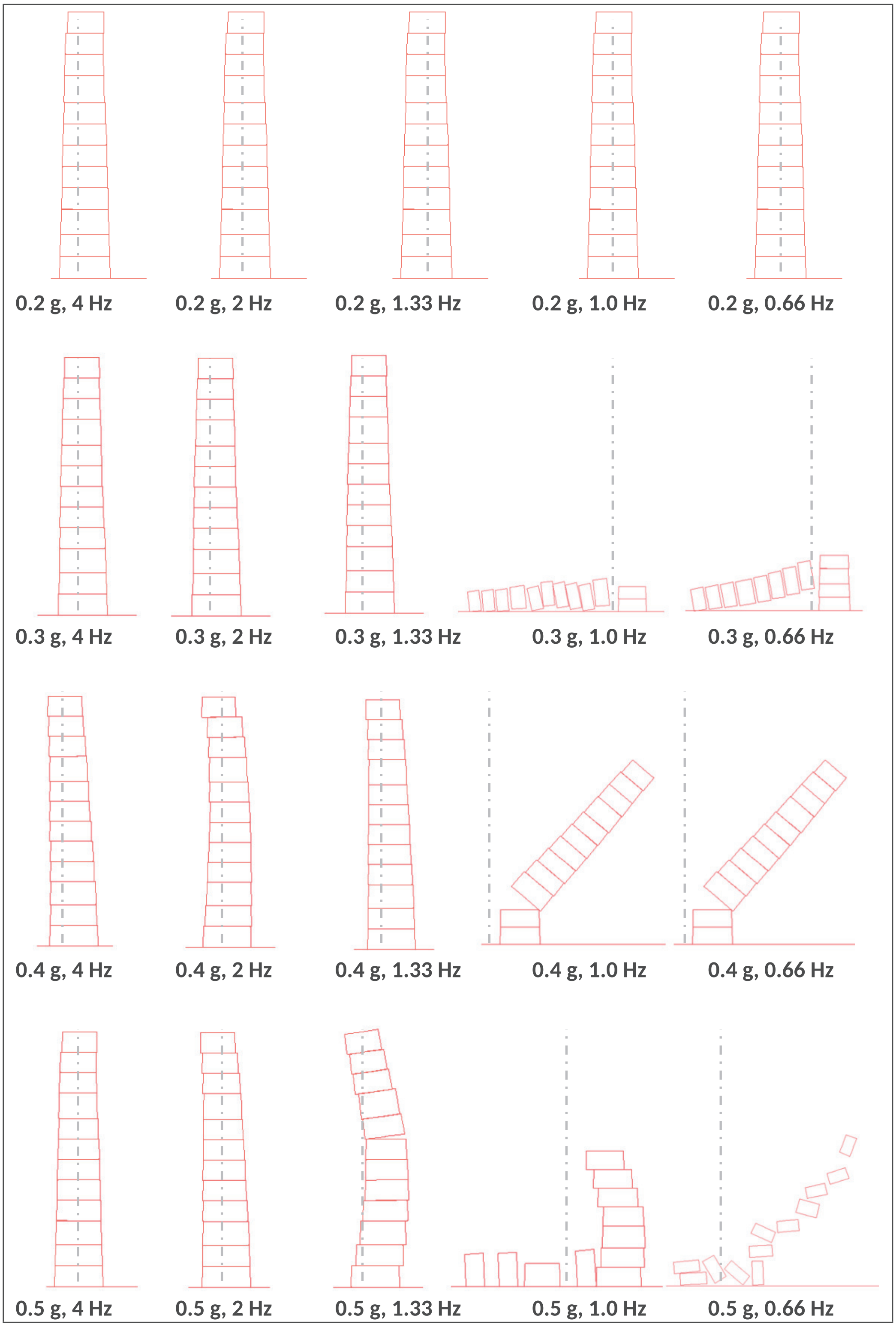

0.2 g, 4 Hz
0.2 g, 2 Hz
0.2 g, 1.33 Hz
0.2 g, 1.0 Hz
0.2 g, 0.66 Hz
0.3 g, 4 Hz
0.3 g, 2 Hz
0.3 g, 1.33 Hz
0.3 g, 1.0 Hz
0.3 g, 0.66 Hz
0.4 g, 4 Hz
0.4 g, 2 Hz
0.4 g, 1.33 Hz
0.4 g, 1.0 Hz
0.4 g, 0.66 Hz
0.5 g, 4 Hz
0.5 g, 2 Hz
0.5 g, 1.33 Hz
0.5 g, 1.0 Hz
0.5 g, 0.66 Hz

Figure 18. Multi-drum column subjected to several combinations of input acceleration magnitude and frequency in horizontal and vertical direction: final deformed shape. Note: dashed line denotes the original centre line position of the drums.

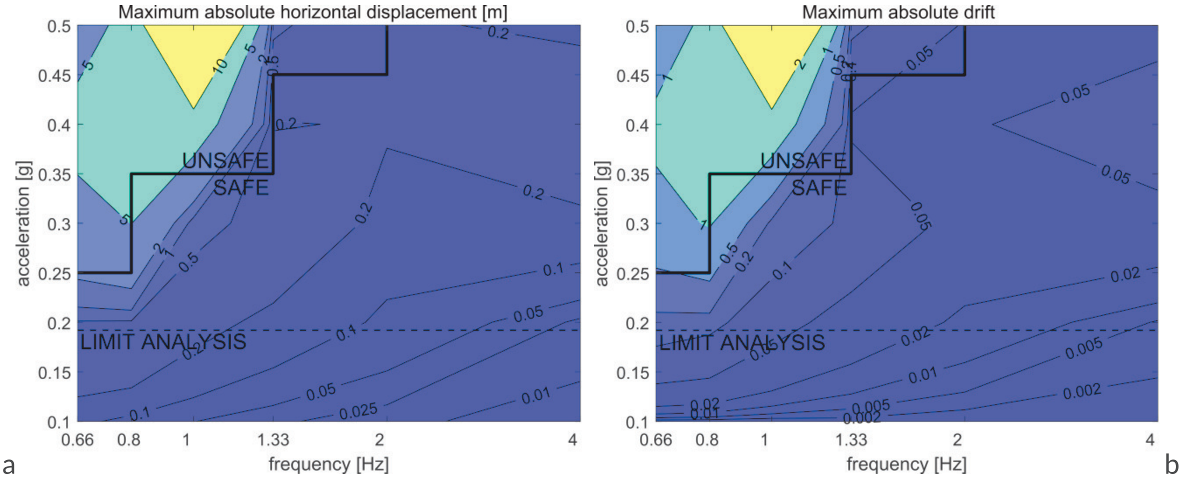
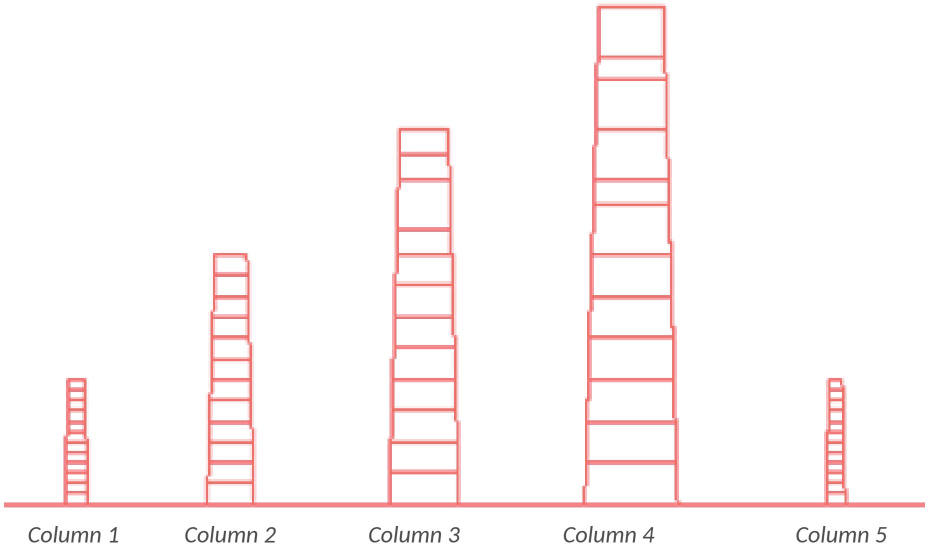


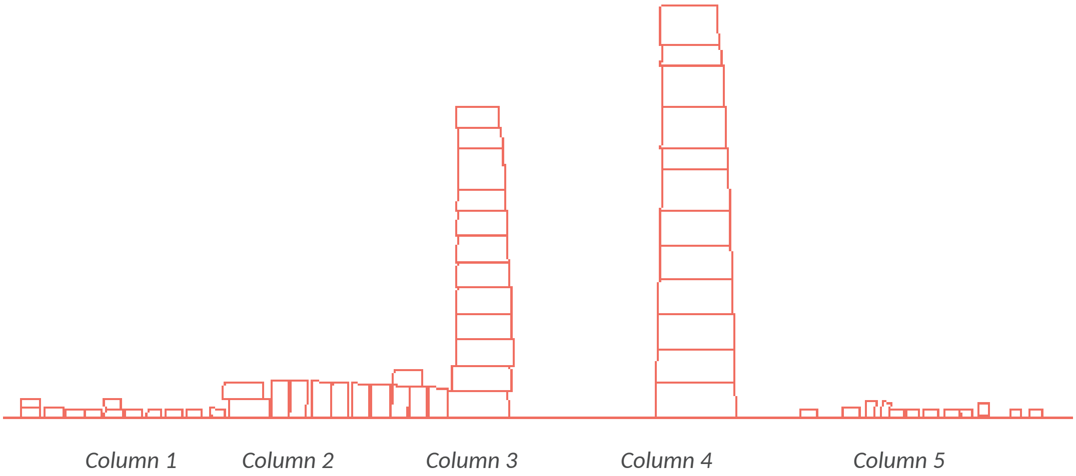
Figure 19. Multi-drum column subjected to harmonic excitation in the horizontal and vertical direction: (a) maximum absolute horizontal and vertical displacement at the top; (b) maximum absolute drift.

4.5 Performance of multi-drum columns closely spaced together

Considering real-life case studies, it is frequent to find historical and/or archaeological sites characterized by columns placed close to each other (see Fig. **12**). In these cases, there can be the possibility for one column to collapse and knock down another one placed next to it, which could have potentially survived an earthquake if it was not interrupted by its neighbouring column. The Fig. **20** below illustrates a series of columns closely spaced together. The size and spacing of the columns has been selected randomly, although such scenario may often be encountered in archaeological sites. The columns have been subjected to a harmonic load with magnitude equal to 0.4 g and frequency 0.66 Hz. Fig. **20** a shows the original configuration of the columns and Fig. **20** b shows their collapse mechanism. It is evident that column 2 disturbed column 3 and created a shear failure mechanism in it. This is something that should be taken into consideration especially when reconstruction purposes are considered. In addition, Fig. **21** shows the displacement of the ground against the displacement at the top left hand corner for each of the five different in geometry columns. Column 1, 2 and 5 collapse and column 2 disrupted column 3 and developed a large displacement at the base of column 4. The large displacement is evident in Fig. **21** at approximately 4.5 seconds where the amount of displacement exceeded one meter in the horizontal direction. The above findings highlight that in the case that there are multi-drum columns closely spaced to together, there is a need to perform dynamic performance of the group of columns rather than of the individual ones since there is a significant potential that collapse of one column could affect the stability of the neighbouring one.



(a) Original shape



(b) Deformed shape

Figure 20. Actual and deformed shape of the multi-drum columns closely placed to each other and subjected to harmonic load in the horizontal direction (a_g = 0.4 g, f = 0.66 Hz)

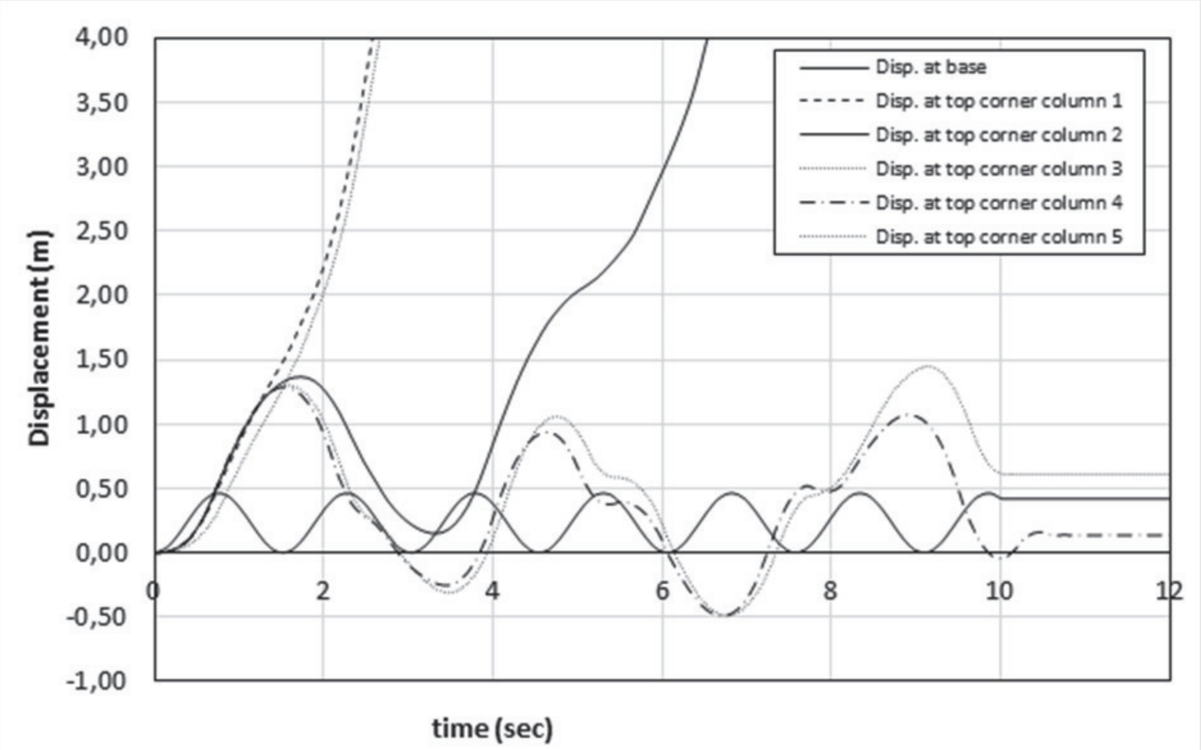


Figure 21. Displacements at the base of the column and displacement at the top corner of the columns against time for the different in dimensions columns, which were placed closely to each other (a_g = 0.4 g, f = 0.66 Hz).

5 Conclusions

There is much to learn from the forgotten architectural and structural principles developed by the ancient builders. Novel structural analysis tools that extend traditional methods could assist engineers to understand the mechanisms that have allowed the surviving structures to avoid structural collapse and destruction during strong earthquakes. By better understanding the seismic performance of ancient structures, better decisions on the conservation and rehabilitation techniques could be made. The aim of this research was to investigate the dynamic response of ancient multi-drum and monolithic columns subjected to horizontal and combined horizontal and vertical harmonic excitations. Using a software based on the Discrete Element Method (DEM) of analysis, a typical ancient multi-drum column and a monolithic one with equivalent dimensions were subjected to harmonic excitations. Such dynamic actions can provide more clear information on the effect of the frequency content of an earthquake to the possibility of collapse with respect to seismic records. Different in magnitude harmonic loads have been applied in: a) horizontal direction; and b) both horizontal and vertical directions. Results from the harmonic analyses were presented in the form of contour maps of maximum column top displacements and drift values for varying input frequency and acceleration magnitude. Finally, harmonic excitations were applied to five different in geometry closely spaced multi-drum columns and their collapse mechanism were evaluated accounting for possible impacts between them.

From the results analyses it was shown that different horizontal excitations applied to a monolithic and an equivalent in size multi-drum column generally confirmed the main findings already obtained by other authors in the last two decades (among the others: Psycharis et al 2000; Dimitri et al. 2001;

Drosos and Anastasopoulos 2014). In particular, the multi-drum column can resist higher levels of base acceleration during high- rather than low frequency excitation. In addition, higher friction angle between the drums results in rocking collapse mode, while lower friction angle between the drums results in shear collapse mode. However, when the column is subjected to horizontal and vertical base excitations, larger horizontal displacements and drift values are obtained with respect to the case when only horizontal base excitation is applied. Collapse mechanisms were characterized by sliding and rocking, with rocking involving both individual and grouped drums. The determination and graphic representation of maximum horizontal top displacements and drifts recorded during the analyses turned out to be an important tool for describing the dynamic behaviour of columns, given that the corresponding contour maps for varying acceleration magnitude and input frequency can represent an alternative to the traditional safe-unsafe domain. Furthermore, maximum displacements and drift values are important in terms of serviceability, given that even if a column does not collapse for overturning or falling drums, namely in case of intermediate or large excitation frequency values, local joint shear failure or base rocking can cause large displacements, which may limit the structural safety of the column. From the contour maps of the column subjected to horizontal excitations and both horizontal and vertical excitations, it was shown that:

- For low frequency (<1.33Hz) and high acceleration amplitude (> 0.35g), the maximum displacement and drift values of the column subjected to horizontal and vertical harmonic excitations were found to be 4 times higher to those when the column was subjected to horizontal harmonic excitation.
- Maximum displacements and drift values corresponding to the safe-unsafe condition were found to be almost comparable for excitation frequencies less than 1 Hz.
- The safe-unsafe threshold was generally found with a maximum drift close to 0.1-0.2 in case of horizontal excitation and close to 0.5 in case of combined horizontal and vertical excitations.
- Considering the safe-unsafe domain obtained for decreasing input frequency, namely converging to a statically applied action, the collapse acceleration converges quite slowly to the well-known overturning limit load multiplier of the column.

Moreover, from the investigation carried out on closely spaced different in geometry and slenderness columns, it was shown that such columns could be more vulnerable to isolated, free standing ones, since they can fail due to impact of the adjacent columns. This should be taken into consideration when reconstruction works are in progress.

It should also be noted that although three-dimensional models of analysis could be better suited to simulate sliding, rocking and wobbling effects of multi-drum columns subjected to seismic excitations, according to Konstantinidis and Makris (2005), two-dimensional analyses could still be used to capture the overall phenomenon and various parameters that affect the dynamic response of multi-drum columns. Therefore, this study concludes that two-dimensional analyses should be used when it is necessary to perform large numbers of simulations to study the effect of various parameters and characteristics. In addition, two-dimensional analysis is much more time efficient and less sensitive to the contact parameters.

In the future, the range of harmonic excitations and acceleration amplitudes could be extended, in order to obtain on one hand more detailed safe-unsafe domains and contour maps of horizontal displacements and drift of the columns and, on the other hand, to better evaluate the convergence to the static load case. In addition, since the majority of presented results and conclusions refer to a single geometric configuration, further parametric studies will be performed accounting for columns having different geometric parameters, such as different column height, slenderness ratio and number of drums. Additional developments of this work will also include the application of real seismic actions to the monolithic and multi-drum columns investigated here, with particular attention to accelerations acting both in horizontal and vertical directions as well as the influence of multi-drum columns closely spaced together.

References

Andreaus U, Casini P (1998) Rocking-sliding of a rigid block: friction influence on free motion. Eng Trans 46,2:143-164.

Aslam M, Godden WG, Scalise DT (1980) Earthquake rocking response of rigid bodies. J Eng Mech Div ASCE 106(2):377-392

Augusti G, Sinopoli A (1992) Modelling the dynamics of large block structures. Meccanica 27:195-211.

Baratta A, Corbi I, Corbi O, Cennamo C, Di Lorenzo C, Orefice R, Voiello G (2006) Rocking of rigid bodies with unilateral constraints. Proceedings of the 5th WSEAS International Conference on Applied Computer Science, Hangzhou, China, April 16-18, 2006 (pp. 676-681).

Croci G (1998a) The conservation and structural restoration of architectural heritage. Advances in Architecture. Computational Mechanics Publications: WIT Press.

Croci G (1998b) The collapse occurred in the Basilica of St. Francis of Assisi and in the Cathedral of Noto. In Roca, Gonzalez, Oñate, Lourenço (Eds.), Structural Analysis of Historical Constructions (pp. 297-317), CIMNE, Barcelona.

Cundall PA (1971) A computer model for simulating progressive large scale movements in blocky rock systems. In Proceedings of the Symposium of the International Society of Rock Mechanics, Nancy (paper II-8). France.

Cundall PA, Hart RD (1992) Numerical modelling of discontinua. Eng Comp 9,2:101-113.
Dimitri R (2009) Stability of Masonry Structures Under Static and Dynamic Loads. PhD Thesis, University of Salento.

Dimitri R, De Lorenzis L, Zavarise G (2011) Numerical study on the dynamic behavior of masonry columns and arches on buttresses with the discrete element method. Eng Struct 33:3172–3188.

Drosos V, Anastasopoulos I (2014) Shaking table testing of multidrum columns and portals. Earthquake Engineering and Structural Dynamics 40,11:1703-1723.

Dorosos V, Anastasopoulos I (2015) Experimental investigation of the seismic response of classical temple columns. Bull Earthquake Eng 13(1):299-310.

Gazetas G. (2018). Discussion on “The effect of sliding on the rocking instability of multi-rigid block assemblies under ground motion” by A.N. Kounadis [Soil Dyn Earthq Eng 104 (2018) 1–14]. Soil Dyn. Earth Engng 104, pp 309.

Giuffrè A. (1991) Letture sulla meccanica delle murature storiche. Eds. Kappa, Rome (in Italian).

Housner GW (1963) The behavior of inverted pendulum structures during earthquakes. Bulletin of Seismological Society of America, 53(1), 403-417.

Ishiyama Y (1982) Motions of rigid bodies and criteria for overturning by earthquake excitations. Earthquake Eng Struct Dyn 10:635-650.

Itasca (2004) UDEC - Universal Distinct Element Code Manual. Theory and Background, Itasca consulting group, Minneapolis, USA.

Komodromos P, Papaloizou L, Polycarpou P (2008) Simulation of the response of ancient columns under harmonic and earthquake excitations. Engineering Structures 30(8):2154-2164.

Konstantinidis D, Makris N (2005) Seismic response analysis of multidrum classical columns. Earthquake Engineering and Structural Dynamics 34(10):1243-1270.

Kounadis A.N. and Papadopoulos G.J. (2016), On the rocking instability of 3-regid block system under ground excitation, Arch App Mech, 86, 957-977.

Kounadis A.N. (2018), The effect of sliding on the rocking instability of multi-ridgid block assemblies under ground motion, Soil Dynamics & Earthquake Engineering, 104, 1-14.

Lemos JV (2007) Discrete element modelling of masonry structures. International Journal of Architectural Heritage 1(2):190-213.

Lenci S, Rega G (2006) A dynamical systems approach to the overturning of rocking blocks. Chaos Solitons & Fractals 28:527-542.

Lignola GP, Giamundo V and Cosenza E. (2014). Finite Element Modelling of the Archaeological Colonnade in Pompeii. Proceedings of the Twelfth International Conference on Computational Structures Technology, B.H.V. Topping and P. Iványi, (Editors), Civil-Comp Press, Stirlingshire, Scotland.

Macchi, G. (1998). Seismic risk and dynamic identification in towers – A Keynote Lecture, Proc. Monument-98, Workshop on Seismic Performance of Monuments, Lisbon, Portugal, K3-K17.

Milne J (1881) Experiments in observational seismology. Trans Seismol Soc Japan 3:12-64.

Mitsopoulou E, Paschalidis V (2001) Numerical evaluation of the seismic response of single-block and multi block monumental columns. In Proceedings of the Second Greek National Conference on Earthquake Engineering and Seismology (pp. 287-296). Thessaloniki, Greece.

Mouzakis HP, Psycharis IN, Papastamatiou DY, Carydis PG, Papantonopoulos C, Zambas C (2002). Experimental investigation of the earthquake response of a model of a marble classical column. Journal of Earthquake Engineering & Structural Dynamics,31(9):1681-1698.

Omori F (1900) Seismic experiments on the fracturing and overturning of columns. Bulletin of the Imperial Earthquake Investigation Committee 4(1):1-31

Omori F (1902) On the overturning and sliding of columns. Publication Earthquake lnvestigation Committee. 12(1):8-27

Papadopoulos K and Vintzileou E (2014). Numerical investigation of the seismic behaviour of ancient columns. 9th International Conference on Structural Analysis of Historical Constructions. F. Peña & M. Chávez (eds.)
Mexico City, Mexico, 14–17 October 2014.

Papaloizou L, Komodromos P (2009) Planar investigation of the seismic response of ancient columns and colonnades with epistyles using a custom-made software. Soil Dynamics and Earthquake Engineering, 29(11-12):1437-1454.

Papaloizou L, Komodromos P (2012) Investigating the seismic response of ancient multi-drum colonnades with two rows of columns using an object-oriented designed software. Advances in Engineering Software, 44(1):136-149.

Papantonopoulos, C., Psycharis, I. N., Papastamatiou, D. Y., Lemos, J. V., Mouzakis, H. P. (2002). Numerical prediction of the earthquake response of classical columns using the distinct element method, Earthquake Engineering & Structural Dynamics, 1(31), 1699-1717.

Pappas, A., Sextos, A., Da Porto, F., & Modena, C. (2017). Efficiency of alternative intensity measures for the seismic assessment of monolithic freestanding columns. Bulletin of Earthquake Engineering, 15(4), 1635-1659.

Pena F, Prieto F, Lourenço PB, Costa AC, Lemos JV (2007) On the dynamics of rocking motion of single rigid-block structures. Earthquake Eng Struct Dyn 36(15):2383-2399.

Pitilakis, K., Tsinidis, G. & Karafagka, S. (2017). Analysis of the seismic behavior of classical multi-drum and monolithic columns. Bulletin of Earthquake Engineering, 15(12), 5281–5307.

Pitilakis K., Roumelioti Z. (2013). The Lefkas M6.2 2003 Earthquake. Vienna Congress on Recent Advances in Earthquake Engineering and Structural Dynamics 2013 (VEESD 2013) C. Adam, R. Heuer, W. Lenhardt & C. Schranz (eds), 28-30 August 2013, Vienna, Austria, Paper No. 383

Psycharis IN, Jennings PC (1983) Rocking of slender rigid bodies allowed to uplift. Earthquake Eng Struct Dyn 11:57-76.

Psycharis IN (1990) Dynamic Behaviour of Rocking Two-Block Assemblies. Earthquake Eng Struct Dyn 19:555-575.

Psycharis, I. N., Papastamatiou, D. Y., Alexandris, A. P. (2000). Parametric investigation of the stability of classical columns under harmonic and earthquake excitations, Earthquake Engineering & Structural Dynamics, 1(29), 1093–109.

Psycharis IN, Fragiadakis M, Stefanou I (2013) Seismic reliability assessment of classical columns subjected to near-fault ground motions. Earthquake Engineering and Structural Dynamics 42(14):2061-2079.

Psycharis, I.N., Lemos, J.V.,Papastamatiou, D.Y., Zambas, C. and Papantonopoulos, C. (2003), “Numerical study of the seismic behaviour of a part of the Parthenon Pronaos”, Earthquake Engineering and Structural Dynamics, 32(13), 2063-2084.

Pulatsu B, Sarhosis V, Bretas E, Nikitas N, Lourenço PB (2017). Non-linear static behaviour of ancient free-standing stone columns. Structures and Buildings 2017, (ePub ahead of Print), 1-13.

Sarhosis V, Asteris P, Wang T, Hu W, Han Y. (2016a) On the stability of colonnade structural systems under static and dynamic loading conditions. Bulletin of Earthquake Engineering 2016, 14(4), 1131-1152.

Sarhosis V, Asteris PG, Mohebkhah A, Xiao J, Wang T. (2016b). Three dimensional modelling of ancient colonnade structural systems subjected to harmonic and seismic loading. Structural Engineering and Mechanics, 60(4), 633-653.

Shenton HW (1996) Criteria for initiation of slide, rock, and slide-rock rigid-body modes. J Eng Mech 122:690-693.

Sinopoli A (1987) Dynamics and impact in a system with unilateral constraints the relevance of dry friction. Meccanica 22:210-215.

Sinopoli A (1991) Dynamic analysis of a stone column excited by a sine wave ground motion. Appl Mech Rev 44(11):S246-S255.

Spanos PD, Koh AS (1984) Rocking of rigid blocks due to harmonic shaking. J Eng Mech 110(11):1627-1642.

Spanos PD, Roussis PC, Politis NPA (2001) Dynamic analysis of stacked rigid blocks. Soil Dyn Earthquake Eng 21:559-578.

Stefanou I, Psycharis IN, Georgopoulos I-O (2011). Dynamic response of reinforced masonry columns in classical monuments. Construction and Building Materials 2011;25: 4325–4337

Yim CS, Chopra AK, Penzien J (1980) Rocking response of rigid blocks to earthquakes. Earthquake Eng Struct Dyn 8:565-587.