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Verso un modello progressivo di graphicacy. Diagrammi cartesiani e ristrutturazione dello spazio cognitivo

Abstract

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La rappresentazione cartesiana dello spazio è un esempio paradigmatico di graphicacy avanzata. Essa richiede non solo l'interpretazione visiva di tratti e simboli, ma soprattutto l'acquisizione di una nuova forma di "alfabetizzazione sinsemica", ovvero una capacità di leggere la composizione di elementi nello spazio grafico.

Tale abilità potrebbe essere comparabile, per complessità e profondità, alla lettura verbale. I diagrammi cartesiani, con la loro struttura bi- o tri-assiale e la semantizzazione quantitativa dello spazio bidimensionale, incarnano un tipo di rappresentazione altamente convenzionale che si discosta radicalmente dalle intuizioni spaziali spontanee dei bambini e di culture prive di istruzione formale.

The Cartesian representation of space stands as a paradigmatic case of advanced graphicacy. It requires more than the visual decoding of lines and symbols; it calls for the development of what might be termed synsemic literacy—a capacity to interpret the spatial organization of graphic elements as a coherent and meaningful structure.

This kind of literacy might be comparable in complexity and cognitive depth to verbal reading. Cartesian diagrams, through their bi- or tri-axial structures and the encoding of quantitative meaning into two-dimensional space, represent a highly conventionalized form of representation—one that diverges markedly from the spontaneous spatial intuitions typically observed in children and in cultures without formal schooling.

1. Reading Cartesian Diagrams and the Hypothesis of Neuronal Recycling

The acquisition of the Cartesian coordinate system represents one of the most profound and least intuitive transitions within the development of mathematical literacy.

Unlike core numerical competencies—which emerge early and are shared cross-culturally, even among populations lacking formal education (Dehaene et al., 2008)—the ability to conceive of space in terms of orthogonal axes, metric proportions, and bidimensional positioning is a distinctly cultural construction.

Although the Cartesian system offers one of the most powerful frameworks for expressing geometric and quantitative relationships, it does not align with innate spatial intuition.

Its mastery requires a progressive sequence of cognitive, semiotic, and instructional mediations—a learning process described by Dehaene and Cohen (2007) as neuronal recycling.

The neuronal recycling hypothesis (Dehaene & Cohen, 2007) posits that culturally recent competencies such as reading or mathematics repurpose evolutionarily older neural circuits, redirecting them toward new functions (Ciccione et al., 2023).

In this light, we may hypothesize that reading a Cartesian graph—which involves the simultaneous decoding of two or three quantities along orthogonal axes, as well as the understanding of scale, intersection points, and the represented function—also results from the reuse of neural systems originally devoted to spatial vision, navigation, and object recognition.

Recent neuroimaging evidence supports this theory. It shows that evaluating trends in scatter plots involves the same occipito-temporal and intraparietal circuits involved in processing object orientation. This indicates a true example of culturally-based neural recycling (Ciccione & Dehaene, 2025) in which visuospatial mechanisms originally dedicated to global shape processing are repurposed for a new function, “visual orientation,” in order to read Cartesian diagrams.

The hypothesis is that learning to interpret coordinate-based representations requires an epistemological shift in spatial understanding: space is no longer perceived merely as a “container of objects,” but rather as a structured field of quantitative variation.

The form of spatial knowledge connected to the Cartesian system emerges, transforms and acquires a distinct epistemic status. This is the result of biologic, evolutionary, ontogenetic and cultural processes that progressively shape the ways in which human beings and the community represent, stabilise and operationalise the concept of Cartesian space (Schemmel, 2016, V, 2-3; Amoretti & Vassallo, 2010).

With Oresme, in the latter half of the fourteenth century, a decisive shift occurred: the spatial grid, initially conceived merely as a device for depicting the local variation of a quality across an extension, was reconceptualized as a genuine space for quantitative representation. Its configurationes systematically separated extension and intensity, allowing intensio to be treated as a measurable, representable and geometrically manipulable quantity. In this way, graphic space ceased to be a container of points and became a domain of variation in which figures, such as the triangle of uniform acceleration, took on a precise mathematical meaning. It was here that the epistemological leap that paved the way for Cartesian space took place: the grid no longer served exclusively to locate, but to express functional relationships between quantities (Schemmel, 2014). This developmental transition involves the internalization of the grid-like structure of Cartesian space, the understanding of the correspondence between points and ordered pairs, and the emergence of metarepresentational skills.

From this perspective, the internalization of the Cartesian grid and the deliberate use of orthogonal reference frames constitute a transformation of the spatial representational system.

According to Bellmund et al. (2018), entorhinal–hippocampal circuits (including grid cells and place cells) may extend their role beyond physical navigation to support abstract "cognitive spaces" structured along conceptual dimensions—including mathematical ones.

he introduction of cultural artifacts such as writing or Cartesian representation might engage neural systems originally involved in visuospatial processing, redirecting them toward new symbolic functions. Mastery of the Cartesian system thus requires refined attention, perceptual reconfiguration, and the integration of vision and symbolization—processes that underpin the development of inferential and representational capacities.

In this process, graphic tools play a central role. Each tool provides a perceptual and conceptual framework that supports the learner in coordinating quantities and relationships within the graphic space.

These tools are not merely illustrative aids, but genuine cognitive vehicles that mediate the transition from pre-structured intuitions to formalized representations.

As several studies have shown (Chumachemko, Shvarts, & Budanov, 2014; Crollen & Noël, 2017; Nardi, 2014), the acquisition of the Cartesian system emerges through a complex interplay between graphical form, cognitive function, and symbolic representation.

Moreover, in the model of cognitive development proposed by Bruner (1964, p. 2), there are three fundamental forms of representation through which individuals learn and internalize concepts: enactive, iconic, and symbolic. In educational contexts, this framework is particularly effective—especially in mathematics and its graphic representations—where the transition from action to symbol mirrors the underlying cognitive trajectory. In light of these considerations, the present contribution aims to offer an integrated analysis of how the acquisition of the Cartesian system unfolds along three main dimensions:

(a) the evolution of the graphic tools employed;
(b) the cognitive abilities activated and transformed;
(c) the semiotic mediations that support the transition from concrete experience to abstract reasoning.

Through a cross-disciplinary reading of neuroscientific, educational, and semiotic literature, a progressive model is outlined—one that seeks not only to describe when and how Cartesian thinking is learned, but also to explain why this form of reasoning constitutes a critical threshold in mathematical development.

2. Cartesian diagrams as a Cultural Construction

Several contributions emerging from the reviewed literature confirm and elaborate the hypothesis that the Cartesian system is a culturally constructed and cognitively mediated representation, consistent with Dehaene's model. Chumachemko et al. (2014) emphasize that learning the Cartesian plane requires a gradual perceptual construction and targeted training in the reading of symbols and spatial relationships. Cartesian systems, in fact, generate a significant cognitive load and must be introduced through targeted instructional strategies.

This load reflects the gap between original numerical intuitions—which tend to follow a logarithmic pattern—and the linear logic of the Cartesian system (Dehaene, 2007). This gap can only be bridged through culturally driven neural specialization.

Crollen and Noël (2017) provide further relevant evidence, showing that while spatial-numerical associations emerge early (e.g., "more to the right = larger"), they become formalized only through structured instruction involving grids, axes, and coordinates.

This distinction between early spatial intuitions and abstract geometric concepts such as proportionality or Cartesian reference is fully consistent with Dehaene's framework.

Finally, Nardi (2014) approaches the issue from a semiotic and geometric perspective, arguing that the use of the Cartesian plane entails a level shift in spatial reasoning—aligned with van Hiele's theory of geometric thinking (1959)—and should therefore be treated as an autonomous language.

This shift is not merely cognitive, but also semiotic: it involves a cultural mediation that integrates perceptual, symbolic, and inferential dimensions, enabling the learner to internalize and master the Cartesian graphic environment.

It can be hypothesized that understanding the Cartesian system involves the cultural repurposing of the dorsal stream of the visual system, which is traditionally responsible for spatial processing in support of action.

Known as the how pathway, this stream encodes egocentric spatial positions, directions, and distances, and is essential for planning goal-directed movements.

In educational contexts, this neurocognitive infrastructure may serve as the perceptual–motor foundation upon which abstract spatial representations—typical of the Cartesian plane—are constructed through cultural and semiotic mediation.

This shift reflects a functional reorganization of the dorsal stream in line with the neuronal recycling hypothesis (Dehaene & Cohen, 2007).

In essence, this process constitutes a conceptualization of space that retools perceptual circuits—originally dedicated to action—in order to sustain advanced forms of mathematical thinking.

This transition from perceived space to represented space, as reconstructed by Schemmel (2016, pp. 9–17), can be situated within an epistemology of space—one in which primary perceptual structures, linked to animal orientation and egocentric mapping, are progressively transformed into culturally shared models culminating in the construction of common reference frames.

Within this framework, the Cartesian plane functions as a synsemic reference frame, capable of catalyzing a reorganization of the entire spatial coding apparatus—transforming the dorsal stream into a neurosemiotic platform for abstract thought.

3. Debate on the role of diagrams

In the theoretical debate on the role of diagrams in mathematical learning, two divergent views emerge regarding their epistemic status and cognitive function. On the one hand, authors such as Silvia De Toffoli (2022, 2023) argue that diagrams are not mere illustrative aids or propaedeutic tools, but rather true epistemic objects: representations that allow us to manipulate, explore and infer in a way that is not reducible to symbolic calculation. From this perspective, the diagram keeps an autonomous function, based on forms of Enhanced Manipulative Imagination (EMI), and conveys content that is constructed through gesture, space and visual perception. This position is based on semiotic and embodied theories, and assumes that mathematical thought is not exhausted in formal language, but is also realised through the sensorimotor mediation of graphic representations.

On the other hand, some studies (Nardi, 2014; Abd Hamid, 2017; Rif'at, Sudiansyah & Imama, 2024) tend to place the diagram in a transitory and subordinate position, useful in the initial phase of learning but destined to be superseded as symbolic skills progress. In these works, the diagram's effectiveness is linked to its ability to facilitate access to formalisation, but its limitations are also emphasised: it may generate ambiguity, increase cognitive load in complex situations or be inadequate to deal with advanced algebraic transformations. Within this framework, the didactic value of diagrams is recognised, but remains anchored to a model of linear progression towards the domain of symbolic-mathematical language.

These two perspectives do not necessarily exclude each other, but they do highlight a fundamental tension in the teaching of mathematics: whether the diagram is a mere scaffolding device to be abandoned, or a full form of thinking that deserves to be cultivated even in the most abstract levels of the discipline. How this tension is addressed has profound consequences for the design of graphical tools, the organisation of learning experiences and the very conception of mathematics as a symbolic language or as an embodied cognitive practice.

In this framework, the term diagram does not denote any visual representation in general, but specifically a graphic device endowed with operational autonomy, in line with the Peircean tradition of diagrammatic reasoning (Dondero, 2010). Following Peirce, the diagram is an icône des relations: a structure that makes abstract relationships visible and manipulable through spatial configurations that can be controlled according to rules that establish commensurability and visual inference.

It is in this rigorous sense that Cartesian x-y diagrams are taken here as a paradigmatic case: the biaxial grid is not a simple support, but an allographic environment that allows functions to be translated into curves, the emergence of mathematical forms (continuity, slope, concavity) to be observed, and visual operations with inferential value to be performed on them. Their strength does not derive from the representation of objects, but from their ability to establish an internal space of transformations governed by rules, in which every modification to the plot produces a coherent change in the conceptual domain. It is this ability to make relationships operational, to stabilise patterns and at the same time keep them manipulable, that defines the Cartesian diagram as an epistemic device: an image with properties of calculation and reasoning, which cannot be reduced to either an illustration or a mere graphic transcription.

4. Cartesian diagrams and the synsemic restructuring of space

The hypothesis that Cartesian coordinate systems involve a deep restructuring of spatial cognition is consistent with the notion of a synsemic dimension of writing (Lussu, 2019)—graphic forms that organise meaning by combining graphic variables (direction, shape, colour, etc.) within visual and spatial relations, which may be governed by patterns or by established habits (Perondi, 2023). The Cartesian plane falls into this category: it is not simply a system of quantitative representation, but a spatial device that restructures the very form of thought.

These synsemic artefacts can be seen in direct relation to the backbone of the visual system. In this sense, Cartesian diagrams would not only be tools for representing quantities, but cognitive devices capable of transforming perceptual space into conceptual space, organised metrically and structured according to inferential relations.

From this perspective, Cartesian diagrams can be understood as cultural artefacts that reorganize visuospatial processing itself, co-opting perceptual mechanisms already shown to support the extraction of global geometric structure, rather than merely depicting quantitative relations, they operate by stabilizing a spatial framework that redirects these mechanisms toward conceptual inference. As explained in chapter 1.1, this hypothesis finds support in recent neuroimaging evidence (Ciccione & Dehaene, 2025).

5. Hypotheses for a progressive model for learning the Cartesian system

In the light of these considerations, we hypothesise that learning the Cartesian system does not simply consist in the acquisition of a graphic technique, but in the progressive internalisation of a cultural form that restructures the cognitive and perceptual space, initiating a process of transition towards a synsemic dimension of representation. This process, of which in this article we propose for simplification an articulation in phases, evolves from spontaneous logarithmic intuitions and is consolidated through the use of graphic artefacts that act as transformative cognitive tools.

In the early phase (pre-graphicacy childhood), the child has a compressed, topological representation of numerical space, supported by circuits in the dorsal stream, which are responsible for visuomotor processing and egocentric navigation. Studies such as that of Crollen and Noël (2017) confirm the existence of an early intuition of number-space correspondence, albeit not yet formalised according to metric criteria, while the work of Dehaene et al. (2008) and Dehaene & Cohen (2007) show that the linearisation of number space is a cultural construction, not an innate one.

In the second stage (early graphic literacy, 6-9 years), the introduction of number lines and linear grids enables the construction of a space with metric properties: concepts such as proportionality and segmentation emerge. Chumachemko, Shvarts & Budanov (2014) illustrate how visual perceptual field patterns help to internalise these structures, while Dehaene (2008) shows that the transition from logarithmic to linear coding depends on educational and cultural factors.

In the third stage (9-13 years), exposure to the Cartesian plane introduces a two-dimensional grid structure that requires the simultaneous coordination of two variables and the activation of meta-representational skills. Nardi (2014) emphasises how mastery of the Cartesian plane coincides with the ability to move from narrative forms to diagrammatic structures, while Gates (2018) highlights the value of diagrams for organising quantitative thinking.

The experimental results of Panavas et al. (2022) confirm that, although children between the ages of 8 and 12 years activate cognitive processes similar to those of adults when reading graphs, their perceptual accuracy is systematically lower in all the conditions analysed. In particular, position on a common axis, length and area are interpreted similarly by both populations, but children make more errors, show greater inter-individual variability and sometimes use divergent strategies.

This finding supports the hypothesis that quantitative graphics, including Cartesian diagrams, is not based on purely perceptual competence, but on a coding system that is refined through exposure, experience and cultural practice. The fact that children and adults share common perceptual bias structures (such as the reliance on cognitive landmarks – e.g. centre, extremes) suggests that the development of graphic competence is based on pre-existing neural mechanisms, consistent with the neuronal recycling model (Dehaene & Cohen, 2007). However, the increased imprecision and variability in children highlights that visual literacy is not automatic and requires a structured educational pathway that accompanies the acquisition of spatial symbolic systems.

These experimental data also indicate that children are not simply "inexperienced adults": they adopt different strategies, sometimes intuitive but ineffective, that reflect different stages of graphic conceptualisation. This reinforces the thesis that understanding

Cartesian graphs requires a conceptual restructuring of space. In the fourth stage (from advanced secondary school onwards), the Cartesian system is consolidated as an autonomous operating environment that can be manipulated through symbolic-visual representations, dynamic software and mathematical models. Chumachemko et al. (2014) show how, at this stage, the coordination between visualisation and symbolic deduction becomes crucial, while Presmeg (2014) and De Toffoli (2022, 2023) defend the full epistemic autonomy of diagrams, emphasising their ability to support abstract thinking through embodied interaction.

From this perspective, the Cartesian plane is a synsemic tool that activates a neural redefinition of space, based on the co-option of the dorsal pathway (Dehaene & Cohen, 2007) and the use of graphic representations capable of transforming perceptual relations into visible and navigable conceptual structures. The Cartesian system thus represents a turning point in graphicacy: a transition in which space becomes a visual linguistic structure.

However, while the progressive model assumes a trajectory towards increasing accuracy and cognitive mastery of Cartesian representations, the results of Xiong et al. (2023) suggest an important counter-trend: even experienced adults may show systematic distortions when reading graphs due to their prior beliefs. Authors show that when the axes of a scatter plot are labelled with semantically "loaded" variables (e.g. pollution and environmental regulation), people tend to overestimate or underestimate the correlation shown based on what they expect or wish to see (Xiong et al., 2023). In contrast, when the same data are presented with neutral axes ("X" and "Y"), the estimates are much closer to the true value.

6. Conclusion

These findings challenge the idea that experience or cognitive maturity guarantee an objective reading of the visualised data, and suggest that graphic literacy, however advanced, remains vulnerable to forms of "motivated perception". It reinforces the urgency of treating the Cartesian plane as a form of epistemological mediation subject to cultural, cognitive and ideological tensions, and raises new questions about the kind of metacognitive training required to ensure a critical and informed reading of quantitative representations.

The path analysed shows how learning the Cartesian system represents a complex cognitive and cultural transformation, simultaneously involving neuro-perceptual structures and graphic practices. The hypothesis of a synsemic transition – according to which Cartesian diagrams act as artefacts capable of restructuring cognitive space and activating a form of organised visual thought – is confirmed in the neuroscientific literature as well as in theoretical and didactic analyses.

The introduction and progressive internalisation of the Cartesian plane is not merely a technical step, but opens up a new mode of thinking, in which space becomes a kind of graphic grammar of graphic grammar–synsemia, precisely.

Cartesian diagrams so intended are complex cultural formations, requiring a deep and progressive synsemic literacy. Acknowledging this nature also implies rethinking educational design: it is necessary to construct experiences that favour the active recycling of perceptive and visual-spatial abilities towards structured, flexible and conceptually powerful forms of representation. The Cartesian system is ultimately only one of the nodal points and most consolidated examples of this transformation in which the graphic dimension of space takes on an epistemic role.

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