Defect and Diffusion Forum
ISSN: 1662-9507, Vol. 419, pp 91-96

Submitted: 2022-07-26
Revised: 2022-07-28
Accepted: 2022-08-01
Online: 2022-10-20

A Mixed FEM for Simulating Laminated Glass Beam Stiffness and Strength

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Keywords: Laminated Glass; elastic interlayer; Mixed Finite Elements; bending tests.

Abstract. This contribution shows the effectiveness of a simple mixed Finite Element Model (FEM) for simulating the mechanical behavior of structural Laminated Glass (LG) beams, by numerically reproducing several laboratory tests of small LG specimens taken from literature. The proposed mixed model adopts small displacements and Euler-Bernoulli beam hypotheses and assumes glass layers displacements and polymeric interlayer shear actions as independent fields. Numerical simulations show that such a simple model is able to correctly reproduce LG specimens' initial stiffness and first peak strength and can be adopted for design purposes of LG elements at a larger scale.

Introduction

Laminated glass is a material originally introduced in mechanical engineering for creating the windshields of aircrafts and automobiles, but its use was rapidly extended to the civil engineering field for creating modern building façades, parapets, transparent roofs or floors and other applications in construction industry [1,2]. LG structural elements can be considered as sandwich or composite elements manufactured by connecting two or more glass layers together by means of one or more transparent thermoplastic interlayers between the layers. LG has essential structural advantages with respect to monolithic glass when the member is loaded up to glass tensile failure, given that monolithic glass has a typical brittle behavior, whereas in a LG structural element the interlayer avoids the detachment of sharp glass fragments after the cracking of at least one of the glass layers, generating a more significant structural safety. The thickness and the type of each glass layer composing the LG element may be equal to or different than each other. This work considers two glass layers made of the same material and having equal thickness, connected by one interlayer. The most common material used for the interlayer is the PolyVinylButyral (PVB) [3].

In this work, a simple mixed Finite Element Model (FEM), recently introduced for modelling LG beam elements with two glass layers connected by an interlayer [4], is further validated with respect to the actual LG behavior determined with laboratory tests. The numerical model assumes small displacements hypothesis and adopts the Euler-Bernoulli beam model for representing the behavior of each glass layer, with the further hypothesis of equal vertical displacements of both glass layers. The FEM is based on a mixed variational formulation that takes into account the displacements of the glass layers and the distributed shear actions of the polymeric interlayer as independent fields. This mixed variational approach leads to a total potential energy of the LG system that is quite similar to that of a standard approach [5] characterized by interlayer slip defined in strong form following Newmark formulation typical of a composite beam model [6]. However, in this case the mixed approach considers each glass layer separately and also accounts for interlayer elastic energy, by defining interlayer slip or shear stress in weak form as function of the horizontal displacements and rotations of upper and lower layer.

A review of the numerical models for representing the behavior of LG beams and plates can be found in the previous work of the author dedicated to the mixed FEM [3], where the model was validated with respect to a standard 2D FEM. Here the model is further and better validated with respect to laboratory results of LG beam specimens subjected to bending, by considering an initial elastic behavior of the materials and accounting for the possible tensile failure of the glass by defining a simple normal force-bending moment strength domain in tension. The model turns out to correctly reproduce actual LG specimens' behavior until the first peak of strength with a very small computational effort if compared with 2D or 3D FEM adopted the authors of the laboratory tests. The final remarks particularly focus on model limitations and its possible future developments, in order to better reproduce the post-cracking behavior of the LG specimens and also to consider different boundary conditions in addition to the simply supported case. This work is also proposed for celebrating the international year of glass [7].

Problem Statement

LG beam. A LG beam with two glass layers connected by a thin interlayer is considered, a two-dimensional (2D) Cartesian coordinate system *Oxy* is adopted, having x coincident with beam longitudinal axis and y directed downward (Fig. 1).

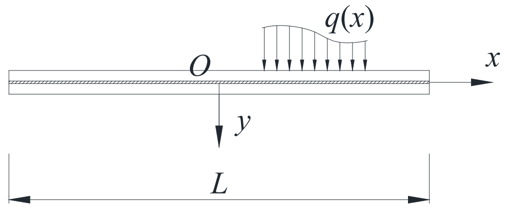
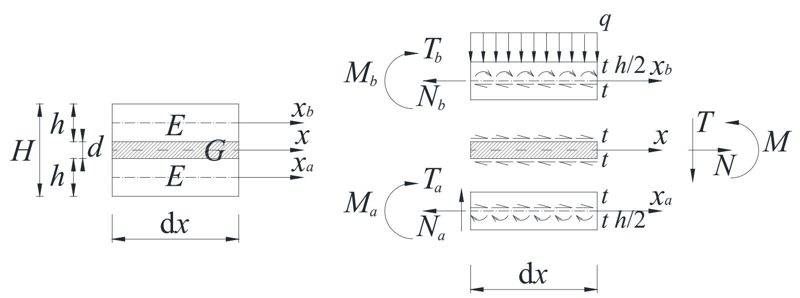


Fig. 1 LG beam.

Upper and lower glass layers are characterized by the same geometrical and mechanical characteristics. Hence, a symmetric beam cross-section having overall thickness equal to $H = 2h+d$ is considered (Fig. 2a), where h is the thickness of each glass layer and d is the thickness of the interlayer. LG beam overall length is L. Plane stress conditions are considered; hence a beam section width B is introduced. LG beam lower layer is indicated with a, whereas upper layer is indicated with b. LG beam layers have the same area $A = Bh$ and moment of inertia $I = Bh^3/12$. Assuming an elastic behavior for the materials, beam mechanical parameters are given by glass elastic modulus E and Poisson's ratio ν, hence the axial stiffness of upper and lower glass layer is EA, and the bending stiffness is EI. The polymeric interlayer is characterized by a shear stiffness k, which depends on interlayer material shear modulus and thickness $k = G/d$, whereas an interlayer infinite normal thickness is assumed in transversal direction, in order to follow the hypothesis of equal vertical displacement for upper and lower glass layers. Focusing also on LG beam initial cracking, the tensile strength f_t of the glass is considered. Such a value is often determined during laboratory campaigns by performing bending tests on monolithic glass elements.



a b

Fig. 2 LG beam infinitesimal element; (a) layer thickness values, (b) internal and external actions.

If the beam is subject to a generic distributed load $q(x)$ on the upper layer, the deformation of the entire element caused by the load generates a distribution of shear actions $t(x)$ along the contact interfaces between the interlayer and upper and lower glass layers. It is worth noting that such distributed forces already consider the uniform distribution of the corresponding normal and shear stresses along beam width. Considering an infinitesimal element of the LG beam having length dx subjected to a distributed vertical load q along the upper layer, the normal, shear and bending actions

(N, T, M, Fig. 2b) at each layer and along the entire beam section must follow the corresponding equilibrium conditions.

Mixed formulation and FE model. Following the past contribution by the author [3], considering each glass layer separately, the total potential energy of the LG beam system may be written by considering separately the contributions of each glass layer and of the interlayer (Π_{intl}). Where the beam elastic energy is given by the contributions of the flexural and the extensional deformations, together with the work of the distributed shear actions acting on the contact surface between the glass layer and the interlayer. The total potential energy of the interlayer can be obtained by considering the Clapeyron theorem and it is given as follows by one half of the shear strain energy over the volume of the interlayer. A mixed formulation is obtained, since the unknowns of the problem are LG beam vertical displacements, cross-section rotations, horizontal displacement of each layer and interlayer shear actions. It is worth noting that this approach allows to correctly simulate the behavior of a LG beam, but also of a generic composite beam, without defining the interlayer slip or shear stress in strong form as function of upper and lower layer displacements and rotations [8].

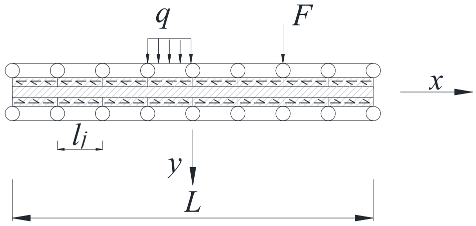


Fig. 3 LG beam subdivided into equal FEs.

Following again the contribution that introduced the mixed FEM for the LG beam [3], a simple discretization of the system is obtained by subdividing the beam into nel equal portions having length $l_j = L/n_{el}$ (Figure 3, with $n_{el} = 8$). Among the different possibilities for discretizing interlayer shear actions, here a constant distribution acting over each glass layer jth sub-element, leading to a piecewise constant distribution of shear actions along the whole LG beam.

Numerical tests and discussion

In this section, several existing case studies are taken into consideration. Several laboratory tests conducted by applying increasing concentrated vertical loads on small LG beam specimens until their failure are reproduced by the proposed mixed FEM. Here, the proposed mixed FEM is able to determine the first level of damage, namely the first cracks appearing along the glass layer subjected to traction, by accounting for the tensile strength of the glass. For this purpose, the numerical approach proposed by Baraldi and Tullini [9] for reinforced concrete frames is considered. Such approach is based on the definition of potential plastic hinges at the ends of the beam FEs, governed by a moment-rotation elastoplastic constitutive law. If the nodal bending moment reaches the corresponding elastic limit, a plastic hinge is activated at the corresponding node and the stiffness matrix of the beam element is modified accordingly. In the following numerical tests, a simple normal force-bending moment domain M_i-N_i, depending on glass tensile strength, is assumed for determining the elastic limit of bending moments. Such a domain assumes a linear relationship between the maximum bending moment without axial force and the maximum normal tensile force without bending moment.

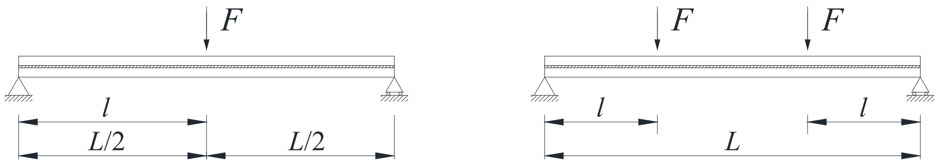


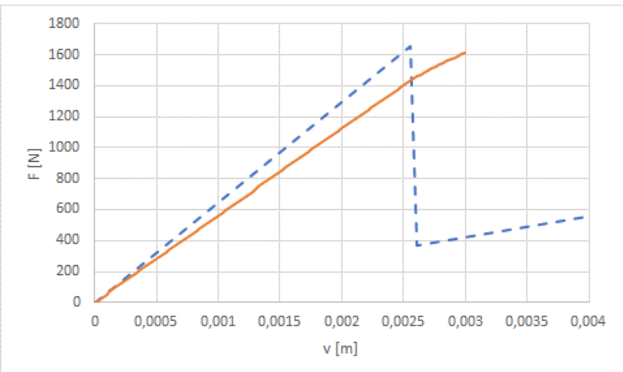
Fig. 4 Schemes of generic three-point (a) and four-point (b) bending tests on a LG beam.

It is worth noting that the pushover analyses are stopped a few steps after the initial cracking, since the proposed model is not able to correctly represent the behavior of a cracked LG beam, in particular, after the first cracking, the hypothesis of coincident bending moments of upper and lower layers is not respected and, furthermore, the interlayer starts to carry tensile stresses, which cannot be simulated by the current numerical model.
The laboratory tests considered are three- or four-points bending tests (Fig. 6a and 6b, respectively) and Tab. 1 collects the geometrical and mechanical parameters of the different specimens. The geometric parameter *l* represents the distance of the applied vertical force with respect to the left support of the beam.
It is worth noting that the interlayer shear modulus *G*, which may be subjected to large value variations due to temperature and velocity of application of the loads [10,11], in some cases it has been estimated by the author if the information was not clearly stated in the corresponding reference, in order to better fit the experimental results. The estimated values of interlayer shear modulus are depicted with an asterisk in Tab. 1.

Table 1. Geometrical and mechanical parameters of the laboratory tests taken as reference.

Authors	L [mm]	l [mm]	B [mm]	d [mm]	h [mm]	E [GPa]	ν	f_t [MPa]	G [MPa]
Akter and Khani (2013) [11] Solutia DG 41	400	200	75	0.76	6.0	70.0	0.23	80.0	45.6*
Akter and Khani (2013) [12] SentryGlass	400	200	75	0.76	6.0	70.0	0.23	80.0	40.5*
Castori and Speranzini (2017) [11] PVB	1000	400	360	0.76	4.0	70.0	0.22	45.0	8.0

Akter and Khani (2013) tests. Among the huge set of laboratory tests performed by Akter and Khani [12], here the three-point bending test performed on three specimens of LG beams with Solutia DG 41 and Sentry Glass interlayers are considered. It is worth noting that the tensile strength adopted in the numerical tests has been slightly reduced (from 92 to 80 MPa) in order to obtain a better fitting of the results. Fig. 5a and 6a show with dashed lines the load displacement curves obtained experimentally, whereas continuous lines represent numerical results, which are in quite good agreement with laboratory ones, with maximum loads close to 1400 N and 1600 N, and corresponding vertical displacements close to 2.5 mm and 3 mm. Fig. 5b and 6b show the normal force-bending moment evaluated at the midpoint of the lower glass layer. In both cases the limit domain is reached close to the end of the numerical analysis and then bending moments are forced to decrease following the domain for increasing normal forces.



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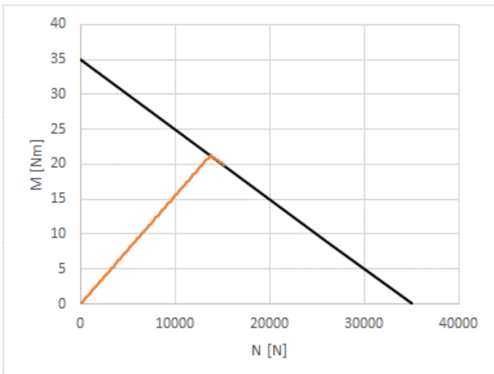

b

Fig. 5 Numerical simulation of the three-point bending test performed by Akter and Khani [12] with Solutia DG 41 interlayer; (a) applied load-vertical displacement at LG beam midpoint, (b) normal force-bending moment variation at lower glass layer midpoint.

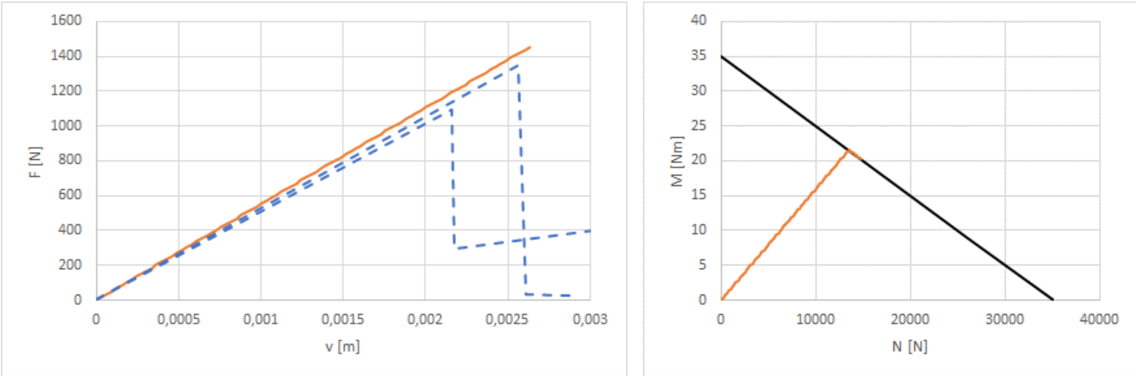

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Fig. 6 Numerical simulation of the three-point bending test performed by Akter and Khani [12] with Sentry Glass interlayer; (a) applied load-vertical displacement at LG beam midpoint, (b) normal force-bending moment variation at lower glass layer midpoint.

Castori and Speranzini (2017) tests. The four-point bending test performed by Castori and Speranzini [13] according to EN 1288-3 on three LG specimens with PVB interlayer are considered. Fig. 7a shows with dashed lines two of the load-displacement curves of three tests performed, whereas the continuous line shows numerical results. The numerical model is characterized by a slightly larger stiffness with respect to the laboratory results; however, the order of magnitude of the maximum vertical load, close to 1000 N, and the corresponding vertical displacement, close to 17 mm, of laboratory and numerical tests is in quite good agreement. Fig. 7b shows normal forces-bending moment curves evaluated along the lower glass layer both at beam midpoint (continuous line) and close to the left loaded node (dashed line).

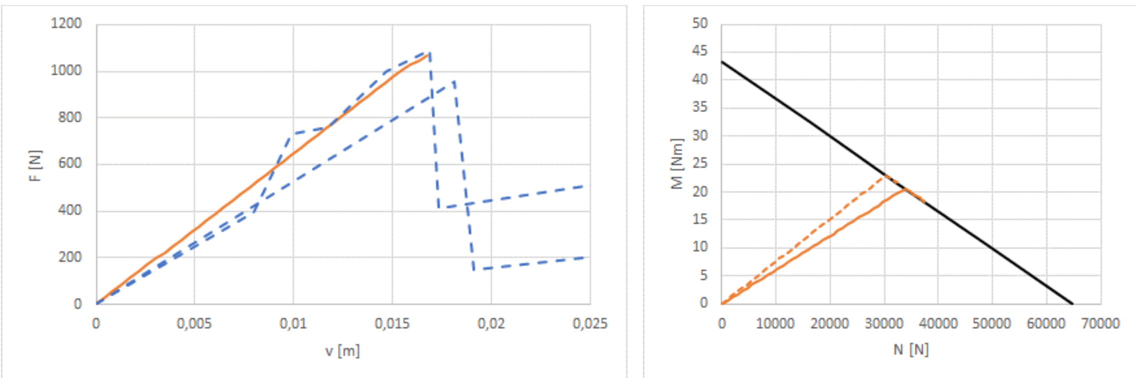

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Fig. 7 Numerical simulation of the four-point bending test performed by Castori and Speranzini [13]; (a) applied load-vertical displacement at LG beam midpoint, (b) normal force-bending moment variation at lower glass layer midpoint (continuous line) and close to the left loaded node (dashed line).

Final Remarks

In this work, a simple and effective mixed FEM, recently introduced for the analysis of LG beams, has been further investigated and validated with respect laboratory experiments. The proposed model, based on the discretization of glass layer displacements and interlayer distributed shear actions, has turned out to be an effective tool for representing LG beam behavior, following the main hypotheses of small displacements, Euler-Bernoulli beam model, equal vertical displacements of glass layers, and no sliding and detachment between each glass layer and the polymeric interlayer. The comparison of the proposed model with respect to three- and four-point bending tests on LG specimens allowed to introduce the initial nonlinear behavior of the glass by accounting for a normal force-bending moment strength domain in tension, based on the tensile strength of the glass. The proposed model has turned out to be in quite good agreement with the laboratory results, both in terms of initial stiffness and first peak cracking load. The model turns out to be a simpler and faster solution with

respect to more complex 2D and 3D FE models, which were adopted by the authors that conducted laboratory tests and can be adopted for design purposes for studying LG beam structural elements at a larger scale with respect to the tests considered.
Further developments of the work, initially considering the elastic behavior of the LG beam, will focus on different boundary conditions of the case studies considered. A comparison with other standard or more accurate FE models usually adopted for composite beams will be also performed.
Other developments of the work, in order to better estimate the nonlinear behavior of the LG beams, will consider the possibility of modelling each glass layers with distinct degrees of freedom in terms of vertical displacements and rotations, given that after the initial cracking of one glass layer, the hypothesis of equal vertical displacements (and consequently equal bending moments) fails to be respected. Furthermore, the capability of the interlayer to carry tensile stresses after the initial cracking will be necessarily taken into consideration. The comparison of this new numerical model with respect to a discrete model for broken laminated glass, already investigated by the author [14], will be performed.

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