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# DYNAMIC ANALYSIS OF MASONRY CHIMNEYS BY MEANS OF A SIMPLE RIGID BEAM MODEL

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## Abstract

*In this contribution, a simple and effective Rigid Beam Model, recently introduced for simulating the dynamic behavior of slender freestanding masonry columns and walls, is adopted and improved for studying masonry chimneys. These structures represent a particular masonry building typology, characterized by a conical shape with a very large slenderness, they are prone to collapse in case of seismic actions due to poor material mechanical characteristics and poor state of conservation. The original model is improved by considering the chimney subdivided into several portions along its height, and each portion is modelled as a rigid beam element with an annular cross-section. Small displacements and no-sliding at the interfaces between the beam elements are the main adopted hypotheses, following the typical assumptions taken by Housner. Material nonlinearity is considered by means of a moment-rotation constitutive law at interface level, also accounting for masonry stiffness and tensile strength. Several numerical tests are performed by considering an existing case study and comparing the modal analysis results of the Rigid Beam Model with respect to those obtained with traditional FE models; then, harmonic tests with varying acceleration and frequency are performed.*

**Keywords:** Masonry, chimneys, Rigid Beam Model, harmonic excitation.

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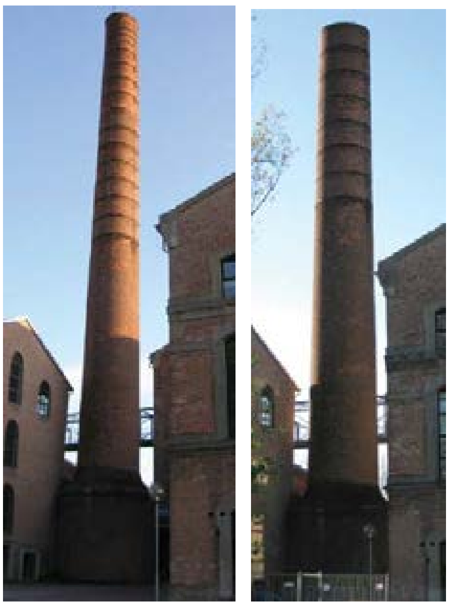
## 1 INTRODUCTION

Unreinforced masonry chimneys are a particular building typology [1], characterized by a truncated-conical shape with a considerable height and a consequent large slenderness. Masonry chimneys were built in most of the industrial countries around the middle of the 19th century, for releasing combustion gases from the industrial plants. Due to the industrial technological improvements, masonry chimneys started to be dismissed in the middle of the 20th century and became part of the huge and various masonry-built environment. However, these structures turned out to be prone to earthquake effects because of their slenderness and their poor state of conservation, mostly motivated by atmospheric agents and due to the limited interest on their restoration. For these reasons, the seismic analysis of unreinforced masonry chimneys is an important challenge in the field of Earthquake Engineering, especially because of the non-standard shape and size of the structure with respect to ordinary monumental and minor masonry buildings.

In this contribution, the original rigid beam model, introduced by authors for modelling the dynamic behavior of monolithic and multi-drum freestanding columns [2] and recently extended for studying cantilever walls [3], is further improved for modelling the dynamic behavior of masonry chimneys. The updated model assumes the chimney subdivided into distinct portions, which are modelled as rigid beam elements with annular cross-section. The model keeps the hypotheses of small displacements and no sliding at interfaces, but it considers masonry mechanical properties for defining interface stiffness and strength into a moment-rotation constitutive law.

An existing case study, represented by a tall masonry chimney located in Ferrara (Italy), is taken into consideration for evaluating the effectiveness of the updated rigid beam model (Figure 1a). This chimney was hit by the Emilia earthquake in May 2012 and after the seismic sequence [4] it was shortened (Figure 1b) by removing an upper damaged portion for safety reasons, since the building is located into the Scientific Campus of University of Ferrara. Several numerical tests have been already performed by adopting accurate three-dimensional models [4,5]; in particular, modal analysis results are here taken as reference for setting the stiffness parameters of the rigid beam model of the chimney. Then, a set of dynamic analyses are performed for determining the safe-unsafe domains of both full and shortened case studies.



a b

Figure 1: Masonry chimney in Ferrara before (a) and after (b) shortening [5].

## 2 RIGID BEAM MODEL FOR MASONRY CHIMNEYS

This work considers a generic masonry chimney with a simplified geometry, having overall height $H$, and characterized by varying outer diameter $D$ and thickness $t$ along the height (Figure 2a,b). The structure is subdivided into $n$ portions having height $h_i$. The rigid beam model is then generated by considering $n$ rigid beam elements and $n$+1 nodes as shown in Figure 2c. Each beam element represents a portion of the chimney and each node represents an interface between the portions. In particular, the first node/interface represents the contact between the foundation and the first portion of the chimney, whereas the last node represents the top of the chimney. Chimney diameter and thickness are defined at each interface level.

Nodal horizontal translational degrees of freedom are considered, namely $u_i$, $\dot{u}_i$, and $\ddot{u}_i$ represent, respectively, nodal horizontal translation, velocity, and acceleration (Figure 2c). Each $i$-th beam element is characterized by a mass $m_i$, which depends on material density $\gamma$ and on the volume of the corresponding portion, which is considered for simplicity as the difference between the volumes of outer and inner cylinders (Figure 2d), assuming an average diameter $\tilde{D}_i$ and an average thickness $\tilde{t}_i$ with respect to upper and lower ones. Due to the rigid beam hypothesis, each element is subjected to a rigid rotation depending on the horizontal translations at beam ends and beam height, $h_i$ (Figure 2e):

$$\theta_i = (u_{i+1} - u_i) / h_i \tag{1}$$

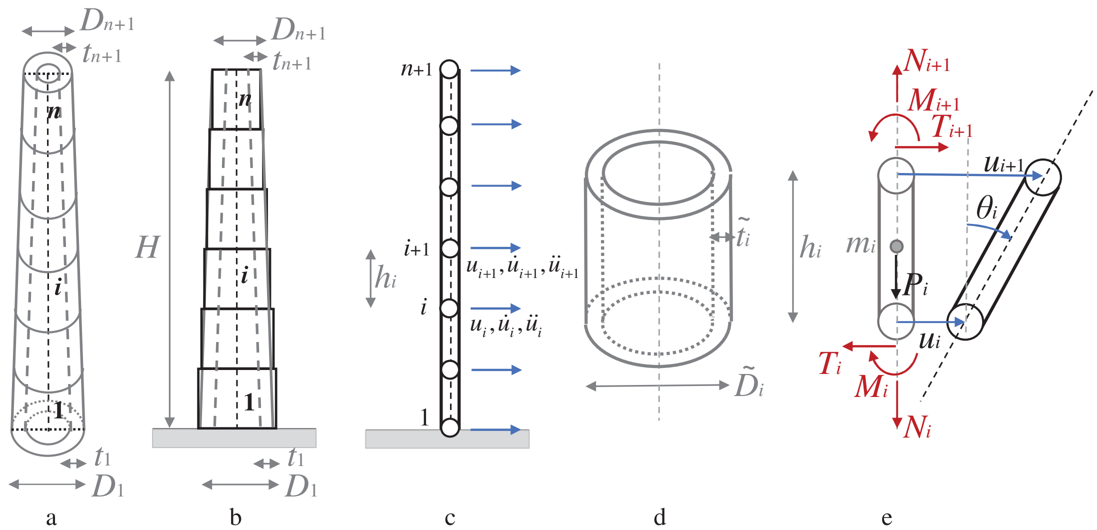



Figure 2: Masonry chimney subdivided into $n$ portions (a), corresponding vertical section (b), corresponding rigid beam model (c), generic chimney portion with average diameter and thickness (d), corresponding rigid beam element (e).

Internal forces of the beam element are given by a normal force $N_i$, a shear force $T_i$, and a bending moment $M_i$, acting at each beam end (Figure 3d).

The translational and rotational equations of motion can be defined with the approach already adopted for multi-drum freestanding columns [2]. If a horizontal ground acceleration $a_g(t)$ acts at the base of the chimney and the top of the chimney is assumed free to move, equations of motion can be written for the entire structure by obtaining the following system of differential equations to be solved:

$$\mathbf{M}(\boldsymbol{\theta}) = \mathbf{G}\mathbf{M}_a\ddot{\mathbf{u}} - \mathbf{G}\mathbf{A}_g + \mathbf{I}_G\ddot{\mathbf{u}} - \mathbf{B}_g \tag{2}$$

Where each bending moment $M_i$ in $\mathbf{M}$ depends on the rotation $\theta_i$ of the corresponding $i$-th drum (Eq. 1). Matrices $\mathbf{M}_a$, $\mathbf{G}$, and $\mathbf{I}_G$ can be called, respectively, mass coefficient matrix, geometric coefficient matrix, and polar inertia coefficient matrix. Details of such matrices and vectors $\mathbf{A}_g$ and $\mathbf{B}_g$ can be found in [3].
The system of differential equations in (2) is solved by means of a Runge-Kutta ODE solver. At this stage, the nonlinear behavior that can affect the chimney is the bending failure at each interface between the portions, whereas, following Housner's hypothesis, shear failure cannot occur [6]. The bending moment $M_i$ at each interface follows a bi- or tri-linear moment-rotation relationship, which represents the maximum stabilizing moment for varying block rotation (Figure 3), and it is slightly modified with respect to Housner's law by means of an initial elastic stiffness $K_{M,i}$ and a smoothing parameter $\xi \leq 1$.

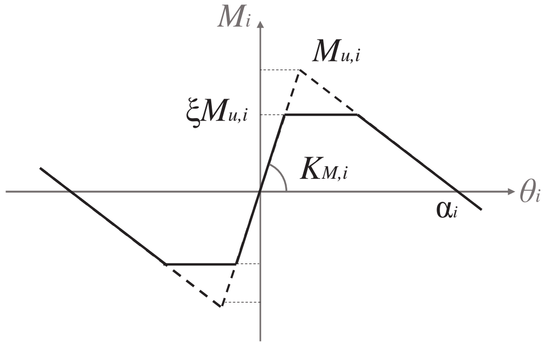



Figure 3: Moment-rotation relationship.

The maximum stabilizing moment in Figure 3 on one hand accounts for the maximum eccentricity of the normal force acting at the $i$-th interface; on the other hand, it also accounts for the tensile strength $F_t$ of the $i$-th interface, depending on masonry tensile strength $f_t$ and chimney cross-section $A_i$:

$$M_{u,i} = \frac{d_i}{2}(N_i + F_{t,i}) = \frac{d_i}{2}\left(\sum_{j=i}^{n} P_j + A_i f_t\right) \quad (3)$$

At the moment, an infinite compressive strength for masonry is assumed, as it was done in the original rigid beam model [2]. Eq. 3 is also adopted into an existing rigid block model for evaluating the bending strength of masonry panel interfaces [7]. However, it can be demonstrated that in case of dead loads close to the 10% of masonry compressive strength, the stabilizing moment evaluated with Eq. 3 is overestimated of about 11% with respect to that accounting for masonry compressive strength.
In the original rigid block model defined for masonry columns or walls, the interface bending stiffness $K_{m,i}$ depends on the elastic modulus of the material $E_m$, on the interface thickness $e_i$ and on the moment of inertia of interface cross-section $J_i$: $K_{m,i} = E_m\, e_i\, J_i$. In case of a masonry chimney, an interface does not actually represent a joint between masonry blocks or drums, hence an interface normal stiffness is introduced: $K_{m,i} = k_m\, J_i$.

## 3 NUMERICAL TESTS

As stated into introduction, the slender masonry chimney located in the old industrial facility that houses the Scientific-Technological Campus of the University of Ferrara, Italy (Figure 1), is taken into consideration for the numerical tests. The full chimney, 50 m high, suffered severe damages during the 2012 Emilia seismic sequence. Afterward, for security reasons, the upper damaged portion, 12.40 m high, was disassembled, leading to a shortened chimney 37.60 m high. Geometric characteristics of the structure are taken from [5] and resumed in

Figure 4a. It is worth noting that the data at only 4 and 3 levels for the full and shortened chimney, respectively, are adopted for the rigid beam model. Mechanical characteristics of masonry for the rigid beam model are material density $\gamma$ = 1800 kg/m$^3$, tensile strength $f_t$ = 0.1 MPa.

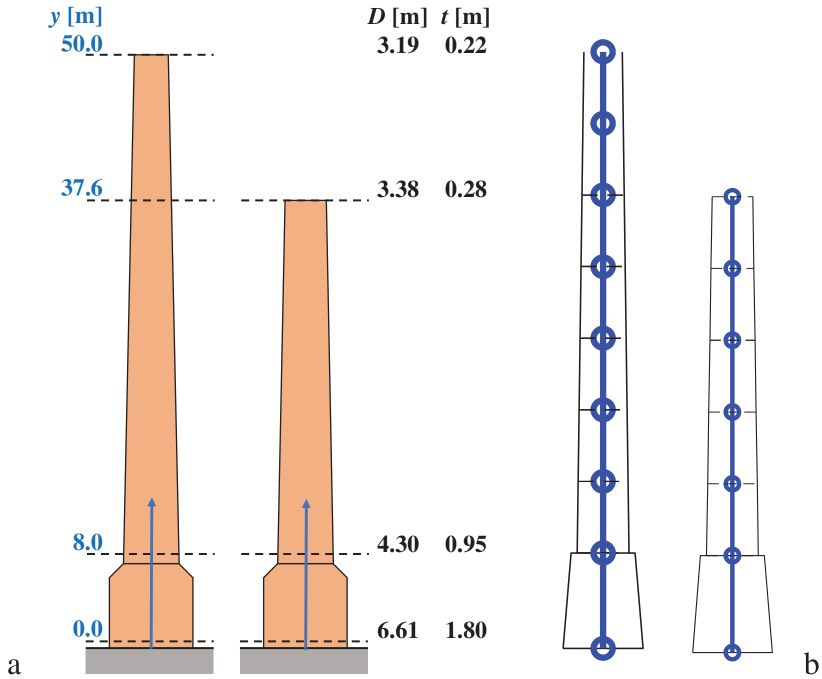



Figure 4: Masonry chimney in Ferrara before and after shortening [5], geometric characteristics (a) and corresponding rigid beam models (b).

Since the chimney is characterized by a thick base from 0 to 8 m, having a larger diameter with respect to the entire chimney, such a base portion is modelled by one rigid beam element. Then, the full chimney from 8 to 50 m is subdivided into 7 equal portions, whereas the shortened chimney from 8 to 37.60 m is subdivided into 5 equal portions, hence by removing the two upper beam elements from the full chimney (Figure 4b). The rotation at the base node of both models is fixed, whereas the stiffness of the interfaces is calibrated by means of a modal analysis of full and shortened chimneys, assuming as reference existing numerical results obtained with an accurate three-dimensional FE model [5].

## 3.1 Modal analysis

| | | frequency | |
|---|---|---|---|
| chimney | model | 1st | 2nd |
| full | rigid beam model | 0.43 Hz | 2.03 Hz |
| | FEM [5] | 0.46 Hz | 2.16 Hz |
| shortened | rigid beam model | 0.93 Hz | - |
| | FEM [5] | 0.85 Hz | - |

Table 1: Comparison between frequencies of the full and shortened chimneys obtained with the proposed rigid beam model and with an accurate 3D FE model [5].

Considering the full chimney, assuming $k_m$ = 800 MPa/mm for all the interfaces of the model, the first and second frequencies obtained with the rigid beam model turn out to be in excellent agreement with existing numerical results obtained with accurate 3D models [5]. Furthermore, the same stiffness parameter applied to the rigid beam model of the shortened chimney allows to obtain at least the first frequency in good agreement with existing numerical results (Table 1).

### 3.2 Dynamic analysis, harmonic excitations

In order to evaluate the dynamic behavior of the full and shortened masonry chimney, with particular attention to their level of safety and to the potential collapse mechanisms that can be activated by dynamic excitations, a set of harmonic tests is performed by varying input frequency and acceleration amplitude at the base of the rigid beam models introduced and calibrated in the previous sub-section.

Starting with the full chimney, Figure 5 shows the deformed configurations obtained at the end of the harmonic tests by assuming several values of base acceleration and input frequency. The corresponding horizontal displacements versus time, evaluated at the top of the full chimney and at the 2nd node (after the thick base) of the rigid beam model are collected in Figure 6. Collapse mechanisms close to the top of the chimney, involving the last or the last two beam elements, are obtained several times: with 0.25g and 0.5 Hz, 0.5g and 1.0 Hz, 0.5g and 2 Hz. In some cases (0.25g with 0.5 Hz and 1.0 Hz) the deformed configurations are also characterized by a significant rigid rotation of a huge portion of the chimney over the thick base.

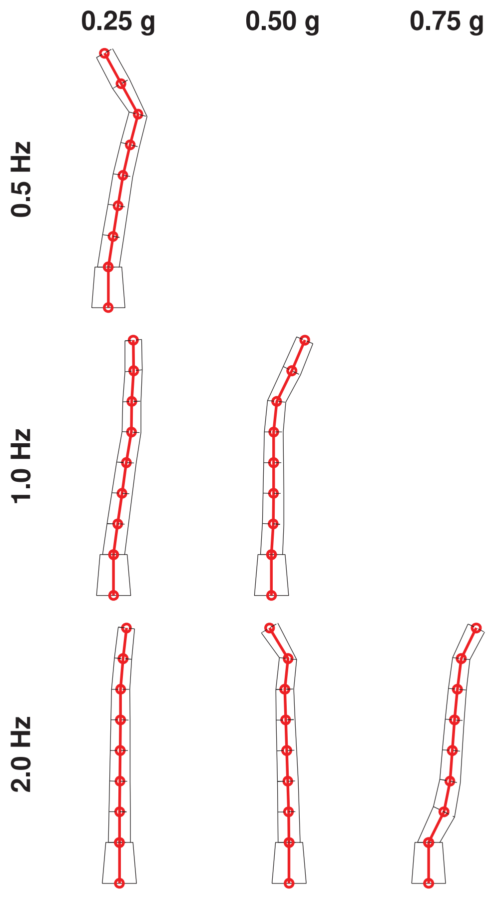



Figure 5: Deformed configurations of the full masonry chimney close to the end of several harmonic excitations.

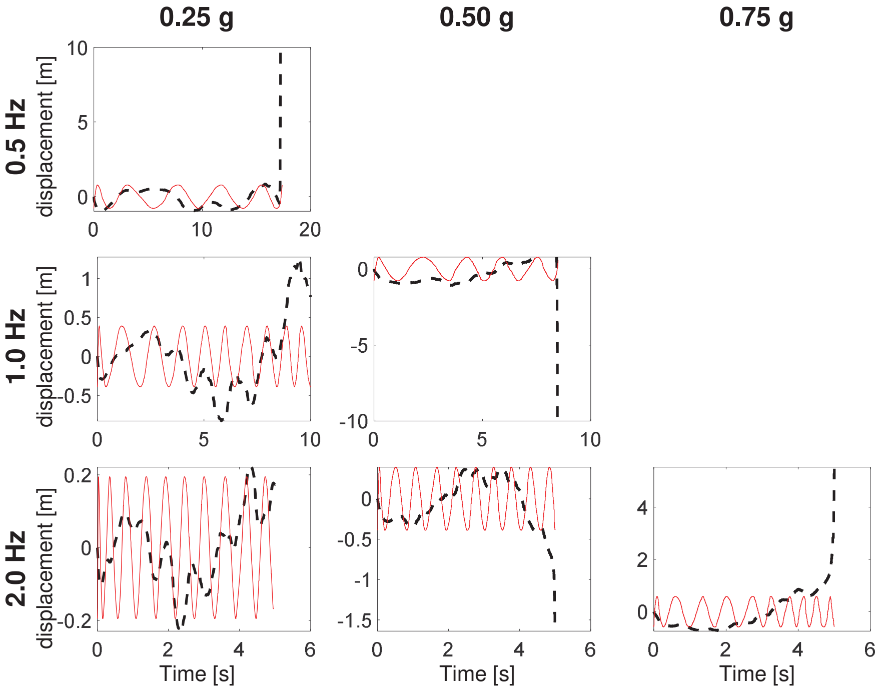



Figure 6: Base (red continuous line) and top (black dashed line) displacements for a of the full masonry chimney subjected to several harmonic excitations.

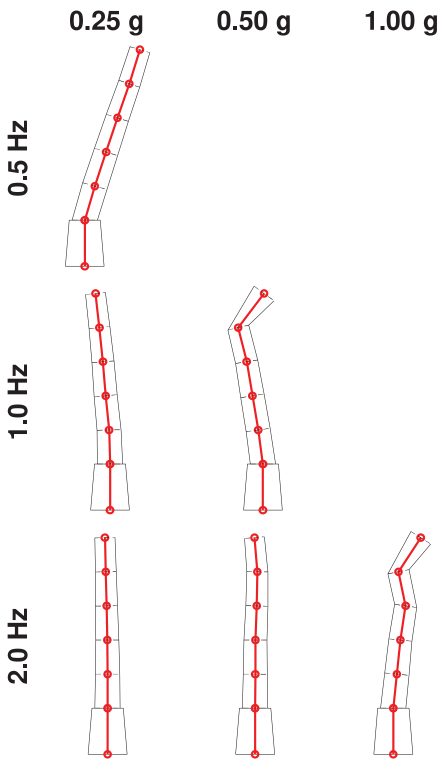



Figure 7: Deformed configurations of the shortened masonry chimney close to the end of several harmonic excitations.

Focusing on the shortened chimney, Figure 7 shows the deformed configurations obtained at the end of the harmonic tests by assuming several values of base acceleration and input frequency. The corresponding horizontal displacements versus time, evaluated at the top of the full chimney and at the 2nd node (after the thick base) of the rigid beam model are collected in Figure 8.

Collapse mechanisms close to the top of the chimney, involving the last beam element, are obtained several times, with larger acceleration amplitudes and frequencies with respect to the full chimney: 0.5g and 1.0 Hz, 1g and 2 Hz. Similarly to the full chimney, in some cases (0.5g with 1.0 Hz) the deformed configurations are also characterized by a significant rigid rotation of a huge portion of the chimney over the thick base. However, with 0.25g and 0.5 Hz, the rigid rotation of the chimney with respect to the thick base involves the entire shortened chimney.

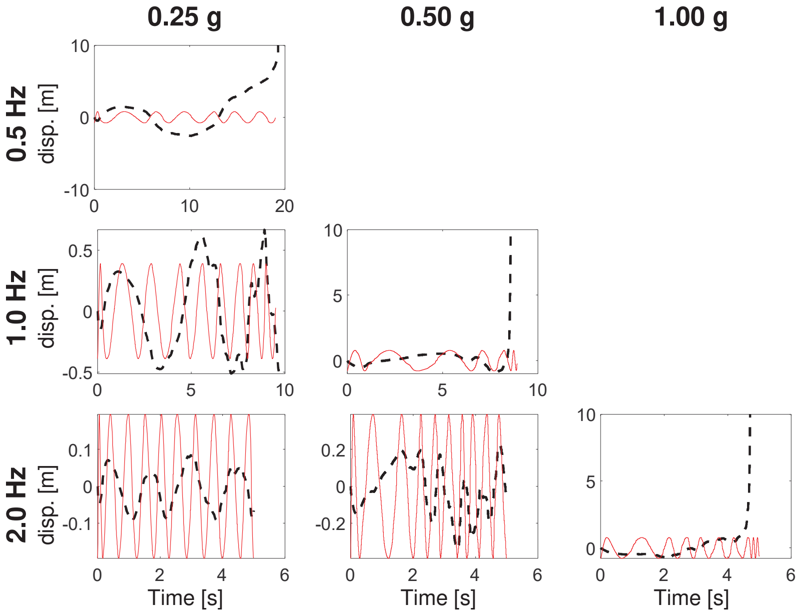



Figure 8: Base (red continuous line) and top (black dashed line) displacements for a of the shortened masonry chimney subjected to several harmonic excitations.

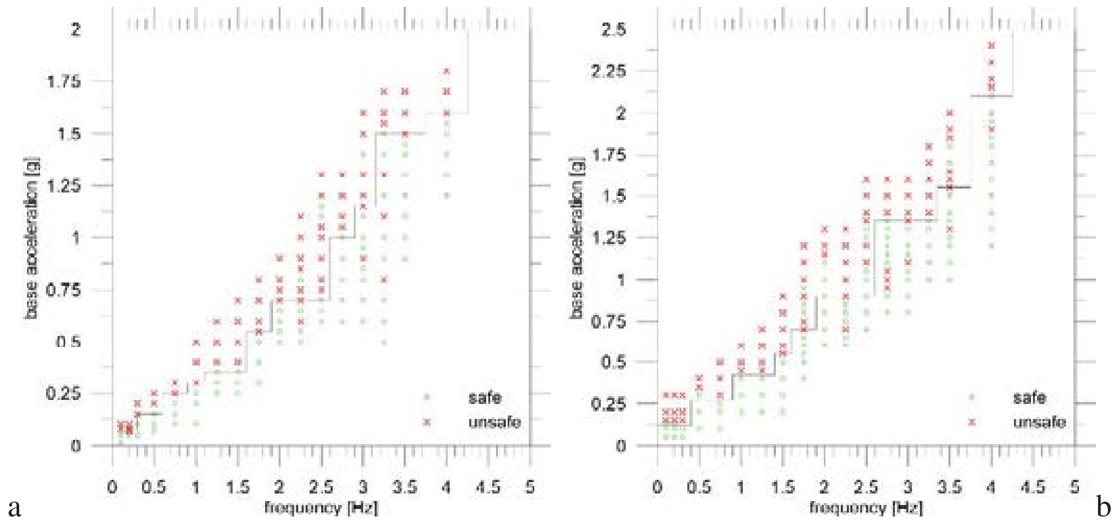



Figure 9: Safe-unsafe domain for the full (a) and shortened (b) chimneys subjected to harmonic excitations.

Figure 9a and b shows the final results of the campaign of numerical simulations on the full and shortened masonry chimneys, by highlighting safe and unsafe conditions at the end of the harmonic tests; a comparison between the two domains is presented in Figure 10. As expected, the shortened chimney is characterized by a larger safe domain with respect to the full one.

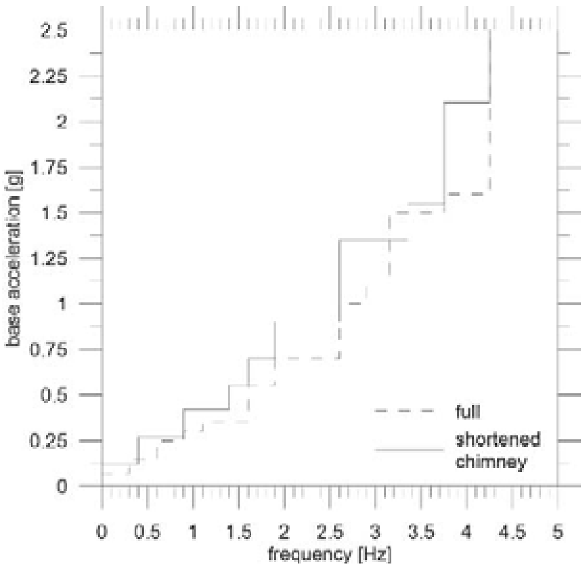



Figure 10: Comparison between the safe-unsafe domains of the full and shortened chimneys subjected to harmonic excitations.

## 4 CONCLUSIONS

This paper presents a simple and effective rigid beam model for studying the dynamic behavior of tall and slender chimneys. The model applied to a real case study i.e. the chimney located in the Scientific Campus of the University of Ferrara which was hit by the Emilia earthquake in May 2012. During the earthquake, the upper part of the chimney was severely damaged. For safety reasons, the chimney shortened (from 50 m which was originally to 37.5 m) by removing its upper damaged portion. In the proposed rigid model, both the original and shortened chimneys investigated. The chimney subdivided into several portions along its height, and each portion was modelled as a rigid beam element with an annular cross-section. In each model, material non-linearity allowed by means of a moment-rotation constitutive law at interfaces, which accounts for masonry stiffness and tensile strength. Initially, modal analysis results were used to obtain the stiffness parameters of the rigid beam model of the chimney. Later, a set of dynamic analysis were performed and the safe-unsafe domains of both full and shorten chimneys obtained. From the results analysis, it is shown:

- A comparison between frequencies of the full and shortened chimneys obtained from the proposed rigid beam model found to be in excellent agreement with existing numerical results obtained from 3D models.
- With respect to the dynamic behaviour of the full height chimney, harmonic excitations with low amplitude (i.e., 0.25g) resulted in rotations and collapse of the upper part of the chimney. On the other hand, harmonic excitations greater or equal to 0.75g caused collapse mechanism of the entire chimney.
- With respect to the dynamic behaviour of the shortened chimney, harmonic excitations with low amplitude (i.e., 0.25g) resulted in rotations and collapse of the upper part of the chimney. The harmonic excitation characterized by 0.75g causes a collapse mechanism involving the entire chimney.
- A comparison between the safe-unsafe domains of the full and shortened undertaken. As expected, the shortened chimney is characterized by a larger safe domain with respect to the full one.

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